

Fat Tails, Illiquidity, and Uncertainty as Explanations of The Credit Spread Puzzle

Gordon Gemmill* and Miriam Marra †

Working Paper
This Version: January 12, 2013

*Warwick Business School, University of Warwick, Coventry, UK (email: G.T.Gemmill@warwick.ac.uk)

†ICMA Centre, Henley Business School, University of Reading, UK (Room 157, ICMA Centre; email: m.marra@icmacentre.ac.uk). This paper was partially written while Miriam Marra was a PhD student in Warwick Business School. The authors thank Prof Anthony Neuberger, Prof Ron Anderson and Prof Ian Marsh for valuable comments and suggestions.

Fat Tails, Illiquidity, and Uncertainty
as Explanations of
The Credit Spread Puzzle

Gordon Gemmill and Miriam Marra
Working Paper

Abstract

Structural models of default risk price firm's equity and debt as contingent claims written on the firm's underlying assets. However, the empirical literature has detected that observed credit spreads, particularly for safer firms, tend to be on average above their structural models' predictions (the *credit spread puzzle*). This paper investigates possible explanations for the credit spread puzzle, using data on the credit default swaps of 40 U.S. investment-grade firms from 2005 to 2009. Firstly, the paper explores the contribution of downside risk, by calibrating the Merton model (1974) on an empirical measure of the sensitivity of credit default swaps to equity volatility, rather than directly on a proxy for asset volatility. The sensitivity measure, extracted from market data, is able to capture the fat left tail in the risk-neutral distribution of firm's returns. Investors take into account the likelihood of extreme events and, to protect themselves against default, they are available to pay a higher CDS premium. Secondly, this work detects two additional components of CDS premia related to illiquidity in credit markets and investors' aversion to uncertainty. The effects of default risk, tail risk, investors' uncertainty, and illiquidity become particularly clear over the recent subprime crisis period when investors *lingered* in fear of crashes and - being uncertain about firms' fundamental values - decided to withdraw from active participation. When market is illiquid and uncertainty greater, sellers of credit default swap charge more and CDS premia increase.

Keywords: Credit Default Swap; Merton Model (1974); Credit Spread Puzzle; Tail Risk; Illiquidity; Uncertainty.

1 Introduction

The spreads on safe (investment-grade) bonds at relatively short maturities tend to be much larger than those predicted by a variety of structural models (Eom, Helwege, and Huang, 2004). Academics and practitioners have defined this phenomenon as the *credit spread puzzle*.

Employing an original methodology, this research paper tests as potential explanations for the credit spread puzzle: (i) the existence of a *tail risk premium*; (ii) investors' *aversion to uncertainty*; (iii) and the *illiquidity* of credit contracts.

The central hypothesis tested is that the levels of credit premia for investment-grade firms above those predicted by the Merton (1974) model are caused by:

- 1) Investors' aversion to the risk of extreme negative events (also called *tail events* or *crashes*);
- 2) Investors' inactivity due to *uncertainty* about firms' fundamentals;
- 3) Lack of market liquidity in the relative credit markets.

In order to estimate the contribution of tail (downside) risk to the price of investment-grade credit, we follow the methodology firstly introduced by Campbell and Taksler (2003) and then employed by Gemmill and Keswani (2011). Instead of using a historical volatility measure for the assets (for example, the one employed by Schaefer and Strebulaev, 2008, or by Vassalou and Xing, 2004), this methodology predicts corporate bond spreads by calibrating the Merton model to the *sensitivity of bond spreads to equity volatility*.

Therefore, we start by estimating the required sensitivity measure from a panel regression of CDS premia on equity volatility variables ($dCDS/d\sigma^E$). The resulting measure captures the pure effects of (market and firm's) equity volatility on CDS premia¹. $dCDS/d\sigma^E$ is then used with the Merton model to obtain weekly implied volatilities and CDS premia for 40 investment-grade firms². Therefore, the calibration of the structural model is neither based on a proxy for the unobservable volatility of firm's asset, nor does it depend on a particular shape of the probability distribution and dynamics of a firm's returns (e.g. with jumps and/or stochastic volatility). The Merton model is instead calibrated on the estimated measure $dCDS/d\sigma^E$ which helps to capture the reaction of investors in the credit markets to shifts in *equity market volatility* and *firm's equity volatility* directly from market data.

Using this methodology we recover implied asset volatilities which display the same *smirk* as shown by the implied volatilities from equity index options³. Previous literature has assessed that this pat-

¹We perform a preliminary control for the simultaneous effects of shifts in firm's leverage and market volatility on both firm's equity volatility and CDS premia in order to reduce potential problems of endogeneity.

²More details on the original Merton (1974) model and its modified version used in the calibration exercise are provided respectively in Appendices B and C. More details on the estimation of $dCDS/d\sigma^E$ are provided in Section 4.

³The implied volatilities from out-of-the-money equity index options are higher than the volatilities implied by the market prices of equity index options which are in-the-money or at-the-money. In cross-section, the implied volatilities of firms with very low leverage ratios are higher than the implied volatilities of firms with relatively larger leverage ratios.

tern in equity options can be explained by the “crashophobia” of investors who are extremely averse to tail events (Jackwerth and Rubinstein, 1996)⁴. Therefore, we call the Merton model calibrated on $dCDS/d\sigma^E$ the “tail-model”.

Some improvements in the estimation of implied asset volatilities and CDS premia are observed in cross-section and time-series using the tail-model rather than traditional (physical) measures of asset volatility. However, the calibration results from the tail-model in and out-of-sample largely depend on the accuracy of $dCDS/d\sigma^E$ estimation and the ability of the panel regression to capture correctly both cross-section and time-series variability in the sensitivity measure.

The use of $dCDS/d\sigma^E$ to calibrate the Merton model reduces the credit spread puzzle, but does not eliminate it. The discrepancies are quite large over the recent crisis period (mid 2007 - 2009). Therefore, we investigate further factors that can help to explain the residual part of credit premia which the tail-model leaves unexplained. In particular, we find that credit spreads are positively affected by *illiquidity* in credit markets and *uncertainty on firm’s future performance and fundamentals*. The economic relevance of these non-default components grows considerably over the crisis period.

Finally, we compare the out-of-sample forecasting ability of the tail-model and its adjusted version (including illiquidity and uncertainty effects) with the forecasts from the Merton model calibrated on traditional proxies for asset volatility, like the volatility proxy used in the 2008 paper by Schaefer and Strebulaev (SS model). The root mean squared forecast error and the mean absolute forecast error from the SS model are double those from the tail-model and triple those from the adjusted tail-model. We therefore obtain the most precise predictions for credit premia using the adjusted tail-model, which takes into account downside risk, illiquidity, and uncertainty premia.

The research paper illustrates how the effects of tail risk, illiquidity of CDS contracts, and investors’ aversion to uncertainty can help to explain (and predict) credit premia for investment-grade firms using the Merton model (1974) framework and some adjustments that take into account the effect of market frictions. The relationships between tail risk, illiquidity, and uncertainty became prominent over the crisis period, when investors *lingered* in fear of crashes and - being uncertain about firms’ fundamental values - decided to withdraw from active participation. When the market is illiquid and uncertainty increases, sellers of credit default swaps charge more and CDS premia widen.

The research paper is written as follows. Section 2 illustrates the main idea and limitations of the Merton model (1974) and explains the intuition behind its calibration to the sensitivity of CDS to equity volatility $dCDS/d\sigma^E$. Section 3 reports preliminary estimates of $dCDS/d\sigma^E$ obtained from: (i) theoretical values of asset volatilities, leverage ratios, and premia predicted by the Merton model; and (ii) real values of equity volatilities and CDS premia used in cointegration analysis on the individual firm level. Section 3 aims to illustrate how $dCDS/d\sigma^E$ estimates change with levels

⁴Traders are concerned about the possibility of a stock market crash similar to the one that materialized in 1987. Thus, they evaluate deep out-of-the-money put options which become very valuable in such extreme scenarios above the Black-Scholes (1973) predictions.

of leverage, asset volatility, and market-wide volatility and provides the basis for the successive analysis. Section 4 explains the procedure followed to obtain the estimated sensitivity $dCDS/d\sigma^E$ from a complete panel regression and the CDS premia and implied volatilities from the tail-model calibration. Sections 5 and 6 show how the tail-model performs in explaining credit premia variability in cross-section and time-series. Section 7 tests the out-of-sample forecasting ability of the tail-model (with and without the extra-premia induced by illiquidity and uncertainty aversion). A robustness check to the tail-model is performed in Section 8. Finally, Section 9 presents the concluding remarks to the work.

2 Beyond the traditional Merton Model (1974)

The Merton model posits a functional relationship between the bond (or CDS) spread S_{it} of a firm i at a specific point of time t and: the asset value (A_{it}), the asset volatility (σ_{it}^A), the leverage (L_{it}), and the time T and interest rate to maturity r_t . For simplicity we can write the explicit form as f_1 :

$$S_{it} = f_1(A_{it}, \sigma_{it}^A, L_{it}, T, r_t) \quad (1)$$

The model's equation contains two unobservable variables: the asset volatility σ_{it}^A and the asset total value A_{it} . A conventional approach to calibrating the model assumes that A_{it} can be approximated by the book value of debt plus the market value of total equity capital. σ_{it}^A is then estimated from the firm's asset value A_{it} , equity value E_{it} and equity volatility σ_{it}^E , and from the first derivative of the firm's equity with respect to its total asset value $\partial E_{it}/\partial A_{it}$, using Ito's Lemma:

$$\sigma_{it}^A \approx \sigma_{it}^E \left(\frac{E_{it}}{A_{it}} \right) \left(\frac{1}{\partial E_{it}/\partial A_{it}} \right) \quad (2)$$

Vassalou and Xing (2004) - hereafter VX - use an iterative process to estimate σ_{it}^A , which has also been followed by other researchers (e.g., Bharath and Shumway, 2004). The iterative process starts by assuming that debt is riskless. In this case, asset volatility is given by:

$$\sigma_{it}^A = \sigma_{it}^E (1 - L_{it}) \quad (3)$$

where L_{it} indicates the firm's leverage ratio. The upper limit for σ_{it}^A is σ_{it}^E (i.e. when debt is risky and debt volatility is equal to equity volatility). The Merton equation (1) is then solved recursively to obtain estimates of the model spread S_{it} .

To estimate σ_{it}^A Schaefer and Strebulaev (2008) - hereafter SS - use instead a combination of σ_{it}^E , σ_{it}^D (respectively the volatility of equity and corporate debt returns), and σ_{it}^{ED} (covariance between debt and equity returns). Using this proxy for the asset volatility, they estimate the debt-to-equity hedge ratio and verify that the Merton model can be used to hedge the credit risk exposure of the

firm correctly, i.e. the model is useful in time-series for hedging of changes in bond prices⁵.

However, most empirical literature that has tested Merton model's ability to predict the *level* of credit spread (rather than changes) has revealed that the model cannot generate the high spreads observed in the markets (see, amongst others, Eom, Helwege and Huang, 2004). These results have originated what is called the *credit spread puzzle*.

Two main streams of literature on the credit spread puzzle have been developed over the past decades. One stream argues that the reason why the classic structural Merton (1974) model tends to under-predict credit spreads is that it relies on over-simplified assumptions on firms' assets dynamics, interest rate dynamics, timing of the defaults, and investors' preferences. The second stream has instead focussed on frictional or non-default components of the spreads on debt and credit instruments. These are components which are not priced in traditional structural models.

2.1 Tail Risk Component in CDS Premia

The Merton model (1974) assumes that the firm's value follows a geometric Brownian motion with constant drift and volatility. In the risk-neutral domain (RND) the firm's returns are assumed to be log-normally distributed. In a structural model for credit risk the holders of risky corporate debt are considered as owners of a riskless bond who have issued put options on the firm value to the holders of the equity. When volatility increases the value of the put option increases and equity holders benefit at the expense of the bond holders.

While the Merton model would predict a positive relationship between equity volatility and credit spreads, Campbell and Taksler (2003) find that the strength of this relationship is far greater than can be explained by the Merton model⁶. This suggests a role for *tail risk* in pricing of credit contracts. An increase in volatility shifts probability mass towards the tails of the distribution of firm value. Economically speaking, a rise in volatility increases the probability of default. However, in the risk-neutral domain asset returns may have a fatter left tail than implied by the log-normal distribution assumed by Merton (1974). Therefore, incorporating *tail risk* (also known as downside risk) in a structural model should lead to better predictions of credit spreads, in particular for investment-grade bonds at short maturities (Delianedis and Geske, 2001; Zhou, 2001b).

Gemmill and Yang (2010) find that the tail risk estimated from equity index options explains credit spreads on zero-coupon bonds. Cremers, Driessen and Maenhout (2008), Zhang, Zhou and Zhu (2009), and Cremers, Driessen, Maenhout and Weinbaum (2008) estimate an option-implied (Q) skewness measure and find some positive evidence of its effect on credit spreads. Coval, Jurek and

⁵Appendix B provides further details on the theory of the Merton (1974) model and on its traditional calibration methodologies.

⁶Different empirical studies detect a positive effect of equity volatility on credit spreads but disagree on its magnitude (see, amongst others, Avramov et al, 2007; Benkert, 2004; Gemmill and Keswani, 2011; Chen, Lesmond and Wei, 2007; and Bharath and Shumway, 2008). Campbell and Taksler (2003) find a huge effect, whereas others find smaller effects.

Stafford (2009), and Collin-Dufresne, Goldstein and Yang (2012) use equity index option prices to explain spreads on triple-A CDOs using a structural model. All these studies show that the structural model can work when suitably calibrated and is capable of generating large credit spreads due to the presence of tail risk premia.

A fat left-tailed distribution can increase credit spreads relative to those implied by the Merton model. However, to get a substantial effect from tail risk, we need to assume that there is a substantial *systematic* or *market-wide* component in the tail event⁷. For this to happen it must be the case that tail events for individual firms can be triggered by default events of other firms via specific business connections or via the “updating of investors’ beliefs” on the likelihood of crashes (see Collin-Dufresne, Goldstein and Helwege, 2003). The tail risk can therefore be decomposed in an idiosyncratic component and a systematic (or market-wide) component (see also Collin-Dufresne, Goldstein and Yang, 2012) that takes into account the possible contagion effect⁸.

There are two possible ways to proceed in order to include tail risk in the structural model: the first is to model the asset return distribution explicitly; the second is to continue to use the Merton model, but to calibrate it differently (depending on the contingent claim being valued) to an empirical measure which can capture the fat left tail and its variability over time.

We follow the second route which keeps all modelling assumptions as simple as in the original Merton model (1974) framework⁹. In particular, in this paper we follow the methodology of Gemmill and Keswani (2011) to predict credit spreads from the Merton model. The methodology requires the calibration of the model to the estimated sensitivity of CDS premia to equity volatility $dCDS/d\sigma^E$ (slope coefficient from a panel regression). This methodology aims to capture the left-skewness (fat left tail) of the firm’s returns distribution implied by real market data. We call the calibrated model the “*tail-model*”.

More specifically, the key idea behind this approach is to recognize that different contingent claims

⁷In the risk-neutral domain, the fat-tailed distribution describes investors’ preferences and it is therefore reflected in all asset prices. In this sense, the tail risk is non-diversifiable and commands a higher risk premium.

⁸The empirical literature has highlighted the fact that often credit default events seem to cluster. Such phenomenon, defined as “*credit contagion*”, depends on the characteristics of the credit event, as well as of the company and the industry (see Jorion and Zhang, 2007). Financial distress across companies may be driven by common economic factors, such as negative shocks to cash flows across the industry or economic recessions. Initially, the literature on the effect of correlated defaults on the individual firm’s default probability started developing with intensity-based models of default risk, rather than with structural models (see, amongst others, Duffie and Garleánu, 2001; Longstaff and Rajan, 2008; and Jarrow and Yu, 2001). In particular, Jarrow and Yu (2001) investigate the effects of counterparty risk, which occurs when the default of one firm causes financial distress on other firms with which the first firm has close business ties. This distress, in turn, is transmitted to a second layer of firms through a domino (or contagion) effect. As examples of this effect, in the 1998 serious concerns emerged that the default of the LTCM hedge-fund would lead to defaults of other major funds and banks. The same concern on default contagion raised with the failure of Lehman Brother in 2008 (amidst the subprime meltdown).

⁹The only modification to the original Merton (1974) model is the additional assumption that at each point of time leverage ratio is equal to the target leverage ratio (see Collin-Dufresne and Goldstein, 2001). Further details are provided in Appendix C.

will trade on different implied volatilities. Risky debt is equivalent to a risk-free bond plus a short position in a put written on the firm value with a strike which reflects the outstanding debt of the firm. Therefore, a CDS written on a firm with very low leverage corresponds to a deeply out-of-the-money put option and has an implied volatility reflecting the mass of the risk-neutral distribution of the firm value in its far left tail. In contrast, a CDS written on a firm with relatively higher leverage corresponds to a closer at-the-money put option, so the implied volatility reflects the mass of the risk-neutral firm-value distribution much closer to the current firm value. As a consequence, by comparing the implied volatilities of firms with lower and higher leverage levels at different points of time we can understand the shape of the risk-neutral distribution of the firm value and how this changes over the time. Instead of imposing a particular volatility or implying the volatility from the observed spreads, which would be the conventional approach, we bypass the choice of a volatility proxy and calibrate the model to the estimated sensitivity of the CDS premia to equity-volatility. If there is a high degree of left-skewness in the risk-neutral distribution, then the calibration of the model on the sensitivity measure will reveal this via implied volatilities which in cross-section are large and diminishing with leverage levels (Gemmill and Keswani, 2011).

2.2 Non-default Components in CDS premia

With regards to the frictional or non-default components of bond and CDS spreads, a number of studies indicate illiquidity as a further explanation for the failure of the structural models (see, for example, Longstaff et al, 2005; Huang and Huang, 2003; and Chen et al, 2007). In our paper we examine, alongside credit market illiquidity, the effect of investors' aversion to uncertainty (also known as ambiguity aversion). Using a structural model for credit risk with heterogeneous beliefs, Buraschi et al (2010) derive testable implications for the role of uncertainty in the determination of equilibrium credit spreads. They show (theoretically and empirically) that there is a positive relationship between the dispersion of investors' beliefs (generated by uncertainty on firm's cash flows valuation and future earnings) and credit spreads. In the cross-section, higher dispersion of investors' beliefs increases firms' credit spreads. In addition, during the 2008 credit crisis the link between uncertainty and credit spreads is found to be stronger than in previous periods¹⁰.

Building on this literature, our research paper reveals the effects of illiquidity and ambiguity aversion on credit spreads which the tail-model cannot capture. Investors' trading decisions and credit pricing are shown to depend also on these frictional components.

¹⁰A more detailed review of studies on liquidity and uncertainty is provided in Chapter 1.

3 Preliminary Analysis of CDS Premia Sensitivity to Equity Volatility

There is within the Merton's model an implicit relationship between the asset volatility (σ_{it}^A) and the sensitivity of the spread to asset volatility ($dCDS_{it}/d\sigma_{it}^A$). Using Equation (2), we can infer that there is also an implicit relationship between the asset volatility (σ_{it}^A) and the sensitivity of the CDS spread to equity volatility ($dCDS_{it}/d\sigma_{it}^E$).

Let us write this as:

$$\sigma_{it}^A = f_2(dCDS_{it}/d\sigma_{it}^E)$$

where f_2 is the implicit function. This function cannot be written explicitly, just as the implied volatility of an option cannot be written explicitly as a function of the relevant parameters. f_2 can only be examined numerically (as we do in Section 4). Next, the CDS premium of firm i at time t can be obtained from the Merton model calibrated to $dCDS_{it}/d\sigma_{it}^E$, rather than to a proxy for σ_{it}^A .

To estimate $dCDS/d\sigma^E$ (i.e. the sensitivity of CDS premia to equity volatility) and capture its variability in cross-section and over time, we employ a panel regression. The calibration results from the tail-model largely depend on the accuracy of $dCDS/d\sigma^E$ estimation. In this Section we collect some preliminary evidence on theoretical and empirical characteristics of $dCDS/d\sigma^E$ to be used then as guidelines for choosing the panel regression specification.

3.1 Estimation of CDS Premia Sensitivity to Equity Volatility from Merton Model based on Theoretical Values of Leverage and Asset Volatility

To specify a relevant panel regression we need to consider what the relationship between CDS premium, leverage, equity volatility, and $dCDS/d\sigma^E$ should be in theory. We assume that there is a CDS with a 5-year maturity written on a firm with asset value of 100 and a continuous interest rate of 5%¹¹. Table 1 displays computed values for CDS premia and $dCDS/d\sigma^E$ under various volatility and leverage conditions, based on a Merton model with fixed leverage ratio.

Starting with Column 1, in the top part of the table leverage is set to 5%; in the middle part it is 15%; and in the bottom part it is 30%. In Column 2, asset volatility is changed in steps of 5%, from 20% to 65%. Column 3 displays the computed CDS premium after Merton-model calibration. Column 4 shows the computed equity volatility, given the level of leverage in Column 1 and the asset volatility in Column 2, using the relationship in Equation (2). In Column 5 of the table, $dCDS/d\sigma^E$ is computed empirically from one row of the table (CDS premia) to the next (equity volatility).

¹¹Whether the asset has any yield or not is irrelevant, because we modify Merton's model so that a firm has a target-level of leverage at the current level. With a fixed leverage, payouts have no impact on CDS values. See Appendix C for details on the model employed and on the calibration procedure.

At low asset volatilities, the spreads from the model (in Column 3) are very small, e.g. 1.8 basis points with a 30% leverage and 20% asset volatility (equivalent to 28.6% equity volatility). However, the model can generate large spreads at even moderate leverage if the asset volatility is large enough. For example, with 15% leverage and an asset volatility of 60%, the equity volatility is 69.6% and the CDS premium is 236 basis points.

For a given leverage, the computed $dCDS/d\sigma^E$ (in Column 5) is quite sensitive to the level of the equity volatility (and associated asset volatility). For example, at a leverage of 15% (which is quite usual for the A-rated firms in our sample) a shift in the equity volatility from 46.9% to 52.6% changes the computed $dCDS/d\sigma^E$ from a value of 3 to a value of 5.3.

The volatilities used in this table are in the risk-neutral “Q” domain and they are not those which can be observed in the physical “P” domain. It is worth mentioning two points in this regard. First, it is well-known that implied volatilities for put options on equity indices show values of 60% or more if the options are deep out-of-the-money¹². A CDS on a highly-rated firm is equivalent to a deep out-of-the-money put option, so it is also likely to exhibit large asset volatility in the Q-domain. Second, in the empirical work in the remainder of the paper we measure equity volatility σ_E in the observable P-domain. We then use σ_E , alongside real data on CDS premia, to estimate $dCDS/d\sigma^E$ in the P-domain. Thus, to conduct any meaningful comparison between the estimated $dCDS/d\sigma^E$ (P-measure) and its theoretical values in Table 1 (Q-measure) we have to assume that $dCDS/d\sigma^E$ is very similar in the P and Q domains. To support this assumption, there is some evidence from equity-index options. The VIX is a measure of equity-index volatility in the Q-domain. Figure 1 shows that the equity-index (S&P 500) volatility measured in P-domain and the VIX index are closely related.

While most previous studies have implicitly assumed that $dCDS/d\sigma^E$ is a constant parameter, the implication from Table 1 is that the theoretical $dCDS/d\sigma^E$ increases with asset volatility (and equity volatility) at each given level of leverage. It also increases with leverage at each given level of asset volatility (and equity volatility), but this second effect is less prominent than the first one. From the two sections at the bottom of Table 1, where leverage is set equal to 15% and 30%, we can calculate for example that if asset volatility doubles from 0.3 to 0.6 *ceteris paribus* $dCDS/d\sigma^E$ becomes on average 20 times larger. Instead, if leverage doubles from 0.15 to 0.3, *ceteris paribus* $dCDS/d\sigma^E$ becomes on average only three times larger. The marginal effect of leverage on $dCDS/d\sigma^E$ is much lower than the marginal effect of asset volatility.

¹²The two reasons for this are: (i) the model assumes a log-normal asset return, whereas it is likely to be skewed; and (ii) the implied volatilities are measured in the risk-neutral (Q) domain.

3.2 Empirical Estimation of CDS Premia Sensitivity to Equity Volatility based on Cointegration Analysis on a Firm-by-Firm basis

Having observed the characteristics of $dCDS/d\sigma^E$ obtained from theoretical values of leverage and asset volatilities using the Merton model, we now use real CDS premia, leverage ratios and realized equity volatilities^{13,14} for 40 investment-grade U.S. firms¹⁵ to estimate $dCDS/d\sigma^E$ on a firm-by-firm basis. These estimates will be then compared with the theoretical values in Table 1.

The estimation of CDS premia sensitivity to equity volatility on a firm-by-firm basis is obtained through cointegration analysis¹⁶. The results are summarized in Table 2. The average $dCDS/d\sigma^E$ estimated over the whole period (Column 1) is positive and in the range between 0.71 and 10.17. The inter-quartile range goes from 1.51 to 3.88, the median is 2.04, and the mean is 3.15. These values are quite consistent with the theoretical values in Table 1.

The range of leverage (from the bottom to the top decile) in our sample of firms goes from 5% to 30%, with a median of 12%. The 5th lowest percentile of leverage is around 5%, while the top 95th percentile is equal to 34%. Assuming a 40% asset volatility (in the Q domain), a range of leverage from 5% to 30% would return from Table 1 a range of $dCDS/d\sigma^E$ from 0.1 to 9.0. At the median leverage of 12%, the computed $dCDS/d\sigma^E$ would be around 3. We conclude that the values of $dCDS/d\sigma^E$ estimated from the cointegration and reported in Table 2 are consistent with the values computed theoretically with the Merton model and reported in Table 1.

Next, Columns 2 and 3 of Table 2 show the cointegration estimates of $dCDS/d\sigma^E$ in the pre-crash period (January 2005 - March 2007) and in the post-crash period (April 2007 - December 2009).

¹³The realized equity volatility (P-measure) is estimated as annualized exponentially-weighted moving average (EWMA) volatility over a 180 days rolling window, using lambda=0.94. RiskMetrics uses generally a lambda of 0.94 for EWMA volatility. The idea behind the EWMA variance is to compute the variance as a moving average of past squared daily returns using decreasing weights. A lambda equal to 0.94 means that the most recent squared daily return is weighted by $(1-0.94) = 6\%$. The next squared return is weighted by the lambda-multiple of the prior weight (5.64%). The third prior day's squared return is weighted by $(1-0.94) \times 0.94 \times 0.94 = 5.30\%$, and so on. Therefore, a value of lambda 0.94 implies a half-life of around 11 days. Recursively, the formula for exponentially weighted moving average variance is given by: $\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1 - \lambda)u_{t-1}^2$ where σ_{t-1}^2 is the previous day's equity variance and u_{t-1}^2 is the previous day's squared equity return.

¹⁴In further extensions of the work implied (Q) volatilities from equity options could also be used as alternative to realized volatilities. The implied volatilities have been used by some authors (e.g. Cremers et al, 2008) to calibrate structural models. This technique requires the knowledge of options prices at different strikes to obtain the implied volatilities of firms with different leverage ratios and then use them to predict credit premia. The estimation of spreads on highly rated firms' bonds requires the availability of very deep out-the-money puts prices. The EMWA measure of firm's realized equity volatility used in our paper is based instead on firm's historical equity returns and it is of easy computation. Furthermore, as mentioned in Paragraph 3.1, the evidence of a close relationship between the VIX index of implied volatility and the realized volatility of the S&P 500 index (see Figure 1) suggest that there may be no substantial difference between $dCDS/d\sigma^E$ estimated with realized equity volatility and $dCDS/d\sigma^E$ estimated with implied equity volatility.

¹⁵See Section 4 for more information about data and relative sources.

¹⁶One reason to use cointegration analysis is that the CDS premium and the equity volatility show high level of auto-correlation, so the underlying relationship between the two variables is difficult to unravel but it is expected to be imposed over time.

Column 2 shows that only 29 out of 40 firms have positive $dCDS/d\sigma^E$ values in the pre-crash period and that the median and mean of $dCDS/d\sigma^E$ are respectively 0.43 and 0.64. Appendix D provides possible explanations for why some firms over this period display negative values of $dCDS/d\sigma^E$. Column 3 of Table 2 shows that in the post-crash period all 40 firms have positive $dCDS/d\sigma^E$ values. In this period $dCDS/d\sigma^E$ has a mean of 3.47 and a median of 2.39. The post-crash results are similar to those estimated for the whole period in Column 1. It would therefore appear that in a quiet period there is little impact of equity volatility on the CDS premia; only in the more turbulent post-crash period does equity volatility have a clear influence.

Furthermore, we test whether the value of $dCDS/d\sigma^E$ estimated on a firm-by-firm basis changes over time with large shifts in general market conditions and volatility. To accomplish this, we perform the cointegration analysis between the firm's CDS premium and its equity volatility, conditional on VIX levels¹⁷. Interesting effects are detected and reported in Table 3. When we move towards higher VIX bands, we observe an almost-monotonic increase in the number of firms displaying a positive $dCDS/d\sigma^E$, and in the mean, median, and maximum value of $dCDS/d\sigma^E$.

Finally, we examine how $dCDS/d\sigma^E$ estimated on a firm-by-firm basis from cointegration analysis changes in cross-section depending on the average firm's leverage and equity volatility, in order to understand whether the patterns are consistent with those observed in Table 1. We find that firms with higher average equity volatility tend to have higher sensitivity of CDS premia to equity volatility¹⁸. The estimated slope from a cross-sectional regression of $dCDS/d\sigma^E$ on firm's equity volatility is positive (equal to 0.17) and highly significant (t-stat is above 3). By contrast, differences in leverage ratios across firms do not seem to dictate significant differences in terms of $dCDS/d\sigma^E$. The estimated slope from a cross-sectional regression of $dCDS/d\sigma^E$ on firm's leverage ratio is negative (equal to -0.39) and insignificant (t-stat is below 1).

The results indicate that: (i) The value of $dCDS/d\sigma^E$ depends on the level of market-wide volatility and increases progressively with higher VIX levels; and (ii) Firms with higher equity volatility also display larger CDS premium sensitivity to equity volatility.

¹⁷The number of observations in each "VIX band" is uneven.

¹⁸The regression results and corresponding graphs are not reported for brevity, but they are available upon request.

4 Estimation of CDS Premia Sensitivity to Equity Volatility from Panel Regression and Calibration of the Tail-Model

We now proceed to estimate the sensitivity of CDS premia to equity volatility $dCDS/d\sigma^E$, to be used in the tail-model calibration, as the slope coefficient from a panel regression of CDS premia on firms' equity volatility and equity market volatility¹⁹. We denote this as the ‘‘auxiliary panel regression’’ since it is instrumental to the model calibration. We have tried a number of specifications for the auxiliary panel regression. Taking into account the theory and the results discussed in Section 3, the specification we favour is a panel regression that allows the value of $dCDS/d\sigma^E$ to vary with the equity volatility level (but not with the leverage level, because the range of leverage is too narrow for the investment-grade firms we consider).

First, we regress the market premium of a CDS contract issued by firm i at time t (CDS_{it}) on the firm's equity volatility (σ_{it}^E), conditional on the level of VIX. The conditionality is formulated as a series of VIX bands, using dummy variables. One reason to use VIX dummy variables is that, as observed in Table 3, the estimated $dCDS/d\sigma^E$ increases in magnitude over time from low to high VIX periods. The panel regression specification aims at capturing these time dynamics. As further explanatory variables we include the equity index volatility ($\sigma_t^{S\&P}$), the interest rate level (R_t), and the firm's leverage ratio (L_{it}).

The resulting regression equation is:

$$CDS_{it} = \alpha_0 + \beta\sigma_t^{S\&P} + \delta_0\sigma_{it}^E + \delta_1 D_{13}^{VIX}\sigma_{it}^E + \delta_2 D_{15}^{VIX}\sigma_{it}^E + \delta_3 D_{20}^{VIX}\sigma_{it}^E + \delta_4 D_{25}^{VIX}\sigma_{it}^E + \delta_5 D_{30}^{VIX}\sigma_{it}^E + \delta_6 D_{50}^{VIX}\sigma_{it}^E + \gamma R_t + \vartheta L_{it} + \varepsilon_{it} \quad (4)$$

D_{13}^{VIX} denotes a dummy variable which equals 1 when VIX is between 13% and 15%; D_{15}^{VIX} denotes a dummy variable which equals 1 when VIX is between 15% and 20%; and so on.

Second, we obtain the sensitivity measure of CDS premia to both *firm's* and *market* equity volatility, $dCDS/d\sigma^E$ as:

$$\frac{dCDS}{d\sigma^E} = \frac{dCDS}{dFirm\ \sigma^E} + \frac{dCDS}{d\sigma^{S\&P}}$$

where $\frac{dCDS}{dFirm\ \sigma^E}$ is the sensitivity of CDS premia to *firm's* equity volatility²⁰ and $\frac{dCDS}{d\sigma^{S\&P}}$ is the sensitivity of CDS premia to *market* equity volatility.

The P-measure of sensitivity of CDS premia to *firm's* equity volatility is obtained from the panel

¹⁹The $dCDS/d\sigma^E$ estimates from cointegration in the previous Section (see Tables 2 and 3) are obtained on a firm-by-firm basis from a time-series analysis which leaves cross-sectional differences unexplained and does not control for the influences of changes in interest rates, market-wide volatility, firms' assets value, and unobservable firm-specific effects on CDS premia. To control for all these effects simultaneously and obtain a pure measure of the sensitivity of CDS premia to equity volatility across all firms and periods we therefore adopt the panel regression approach.

²⁰ $Firm\ \sigma_{it}^E$ is obtained as the orthogonal component of a firm's equity volatility to its leverage L_{it} and to market volatility $\sigma_t^{S\&P}$.

regression as:

$$\frac{dCDS}{dFirm\ \sigma^E} = \delta_0 + \delta_1 D_{13}^{VIX} + \delta_2 D_{15}^{VIX} + \delta_3 D_{20}^{VIX} + \delta_4 D_{25}^{VIX} + \delta_5 D_{30}^{VIX} + \delta_6 D_{50}^{VIX} \quad (5)$$

To this value we add the estimated coefficient for S&P500 equity index volatility from the panel equation (4) $\frac{dCDS}{d\sigma^{S\&P}} = \beta$, which takes into account the effect of general market volatility on CDS premia. Investors' appreciation of a firm's downside risk depends in fact on both the level of riskiness of the firm and shifts in market-wide conditions. We finally obtain:

$$\frac{dCDS}{d\sigma^E} = \delta_0 + \delta_1 D_{13}^{VIX} + \delta_2 D_{15}^{VIX} + \delta_3 D_{20}^{VIX} + \delta_4 D_{25}^{VIX} + \delta_5 D_{30}^{VIX} + \delta_6 D_{50}^{VIX} + \beta \quad (6)$$

We analyse 40 investment-grade U.S. firms over the period from January 2005 to December 2009. To ensure availability of CDS data, we use the quotes of firms that have been listed in the CDX North American Investment Grade Index. We eliminate all financial, insurance, and real estate firms due to the difficulty of interpreting their capital structure variables. Equity and CDS data (prices and quotes) are collected respectively from CRSP and Bloomberg databases. The 1st and 99th percentiles of CDS and equity quotes and returns are winsorized in order to eliminate potential outliers.

We focus our study on U.S. dollar-denominated CDS contracts with five years maturity, as they are the most liquid. Compared to the yield spreads of corporate bonds, CDS spreads are often regarded as a superior measure of default risk as they are not affected by the choice of the risk-free yield or by differential tax treatments. Our sample covers the period that goes from January 2005 until December 2009. Although most information in this study is available on a daily basis, we prefer to use weekly data for several reasons. As noted by Zhang, Zhou, and Zhu (2009), using daily data on single-name CDSs may result in substantial sparseness problems, especially in the very early sample period (2005). Moreover, using daily data is likely to understate the effect of firm's leverage on CDS premia since the balance sheet information (taken from COMPUSTAT) is available only on a quarterly basis. Finally, the impact of outliers and possible measurement errors, for example due to CDS bid-ask spreads, is likely to be much lower for weekly data. Our final weekly filtered dataset contains prices and quotes for equity and CDSs of 40 U.S. investment-grade firms from 1 January 2005 to 27 December 2009, giving a total of 10280 observations. The sample includes the turbulent period of the crisis from mid 2007 to the first quarter of 2009.

The P-measure of CDS premia sensitivity to equity volatility $dCDS/d\sigma^E$ is obtained as in (6) from the panel regression in Equation (4). Since it is difficult to separate the effects of leverage, market volatility, and firm's equity volatility²¹, we employ a two-stage panel regression to estimate Equation (4). In the first stage, equity volatility (proxied by the firm's annualized exponentially-weighted moving average EWMA equity volatility) is regressed on market volatility (proxied by S&P500 annualized EWMA volatility) and leverage ratio. The residuals (defined "equity orthogonal volatility") are then used in the second stage regression. In this second stage, CDS premia are regressed on

²¹As illustrated also in Appendix D, changes in leverage have simultaneous effects on CDS premia and assets volatility (and equity volatility). Therefore, it is important to control for both effects and to disentangle their separate components.

interest rate (proxied by the 5-year Treasury yield at constant maturity), leverage ratio, “orthogonal equity volatility”, “orthogonal equity volatility” multiplied by “VIX bands dummies”, and market volatility. With the second stage regression we can identify the separate effects of leverage, firm’s equity volatility (via orthogonal equity volatility), and market-wide volatility on CDS premia, while avoiding collinearity.

The second-stage estimation results are presented in Table 4. We have estimated Equation (4) with and without fixed firm effects and the results are similar, so we only report the latter in Table 4. The analysis shows that the selected variables are all highly significant in explaining the level of CDS premia. The volatility variables and leverage have positive effects on CDS spreads, while the interest rate has a negative impact. The adjusted R^2 is around 57%. The estimated $dCDS/d\sigma^E$ is equal to $(1.94 + 0.71=)$ 2.65 when VIX is below 13%. This level rises to $(2.65 + 4.59=)$ 7.24 when VIX is in the range of 30 - 50% and remains at a similar value of $(2.65 + 4.50=)$ 7.15 at even higher VIX levels. The estimated $dCDS/d\sigma^E$ is an increasing step-function of VIX (the higher the VIX, the higher $dCDS/d\sigma^E$). Figure 2 shows that the estimated $dCDS/d\sigma^E$, which rises stepwise with VIX in the regression, can be fitted to a logarithmic function of VIX.

The results of the auxiliary panel regression in Table 4 are consistent with both the theoretical values of $dCDS/d\sigma^E$ in Table 1 and the values estimated on a firm-by-firm basis via cointegration analysis (Tables 2 and 3). The mean of $dCDS/d\sigma^E$ estimated from the auxiliary panel regression is 4.45; the median is 3.99; the maximum value is 9.14; the minimum value is 2.03; and the standard deviation is 1.66²².

After having estimated $dCDS/d\sigma^E$, we use the results to calibrate the Merton model. To implement the Merton model using data on 5-year credit default swaps we make two assumptions: (i) the CDS spread is the same as the spread on a risky zero-coupon bond with maturity equal to the CDS 5-year maturity; and (ii) the firm’s current leverage ratio is also the firm’s target level ratio (see Collin-Dufresne and Goldstein, 2001). Details on the model assumptions and the calibration procedure are provided in Appendix C. For each firm and each week the calibration exercise returns values for the firm’s implied asset volatility and CDS premium. We call this model the “tail-model”²³. The reason for calling it the “tail model” is that the calibration to $dCDS/d\sigma^E$ captures an important feature in the data. As illustrated in the next Section 5, the implied volatility smile obtained from the cross-section of model-estimated CDS premia - tantamount to the smile of deep out-of-the-money put options - at each point of time declines with the leverage of the firm (i.e. as the strike price for the put option written on the firm’s assets rises). The equity option literature suggests that this feature is motivated by a heavy negative skew (fat left tail) in the risk-neutral probability distribution of the underlying asset²⁴.

²²The sensitivity measure $dCDS/d\sigma^E$ obtained from the auxiliary panel regression (4) includes both CDS premia sensitivity to firm’s (orthogonal) equity volatility and CDS premia sensitivity to equity index volatility. The latter is approximately equal to 2.

²³Tail-model errors are then generated as difference between: (i) observed market CDS premia and tail-model implied CDS premia; and (ii) asset volatilities implied by CDS market premia and volatilities implied by the estimated tail-model.

²⁴For a put equity option the underlying is the firm’s equity, while for the CDS the underlying is the overall firm’s assets

5 Cross-Sectional Fit of the Tail-Model

After calibrating the tail-model, we analyse its cross-sectional fit by comparing: (i) market CDS premia with the premia estimated from the tail-model, and (ii) implied asset volatilities from market CDS premia with the implied asset volatilities estimated from the tail-model. Figures 3, 4, and 5 show the results for the relatively calm week starting on 19th of March 2006; Figures 6, 7, and 8 show the results for the week starting on the 21st of September 2008, soon after the Lehman Brothers crash.

The interesting evidence displayed by Figures 3 and 6 is that both implied asset volatilities from CDS market premia and from the tail-model decrease with higher leverage ratios and remain very close, even during the turbulent week after the Lehman collapse (21 September 2008). Implied asset volatilities decrease when the leverage ratio increases, just as implied volatilities from deep out-of-the-money put options decrease with the strike price. This confirms our hypothesis that $dCDS/d\sigma^E$ is able to capture non-normality and left-skewness in the risk-neutral distribution of a firm's returns.

While leverage has a decreasing effect on implied volatilities in cross-section, this does not always translate in a negative effect on CDS premia. In fact in Figure 5 CDS premia are only slightly decreasing with leverage, while in Figure 8 they are slightly increasing with leverage²⁵.

In Figures 4 and 7 we also compare in cross-section the implied asset volatilities from CDS market premia with two P-measures of historical asset volatility, including the one estimated using the Schaefer and Strebulaev (2008) methodology (SS). Figures 4 and 7 present interesting differences with respect to Figures 3 and 6. The volatility estimates from the tail-model have a pattern which is much closer to their market-implied values and in this they outperform the SS estimates. The tail-model also outperforms the Merton model calibrated to SS volatility (we call it SS model for brevity) in predicting CDS premia (see Figures 5 and 8). In the week starting on the 19th of March 2006, the average prediction error for the tail-model is 15 basis points (equivalent to 50% the average market CDS premium). This error is half the average prediction error from the SS model (around 29 basis points). The SS model works extremely badly over the cross-section. In the week starting on the 21st of September 2008, the average prediction error for the tail-model is 13 basis points (equivalent to 18% the average market CDS premium), which is less than half the SS model average prediction error (34 basis points, corresponding to almost 50% of the average market CDS premium).

The in-sample results show that even a simple Merton model can have more explanatory ability when calibrated on a measure that proxies left skewness in the risk-neutral distribution of firms' returns (and investors' appreciation of extreme downside events). However, we observe that the CDS premia estimated from the tail-model remain on average lower than the CDS premia observed in the market. We therefore analyse next whether the in-sample tail-model prediction errors can be explained by

value.

²⁵The estimated coefficient of leverage in the auxiliary panel regression in Equation (4) is also found positive (see Table 4). Appendix D explains the consistency of these results in presence of different cross-sectional and time-series effects of leverage and volatility on CDS premia.

non-default factors, such as illiquidity and investors’ aversion to uncertainty.

The illiquidity of the CDS contracts can increase CDS premia above the level predicted by the structural model. Recent papers, such as Tang and Yan (2006) and Bongaerts, de Jong and Driessen (2011), have studied the effects of liquidity on derivative contracts in zero net-supply, particularly on CDS contracts. Tang and Yan (2006) observe that “sellers of CDS contracts provide not only insurance against credit risk, but also liquidity service in the market. If the demand exceeds the supply, sellers can charge a premium for faster matching, *ceteris paribus*, as they are liquidity providers”²⁶. Bongaerts, de Jong, and Driessen (2011) demonstrate that illiquid derivatives can have lower expected returns if the short-sellers (CDS protection writers) are more aggressive than the “long” investors (CDS protection buyers), due to their higher aggregate wealth, lower risk aversion, or shorter horizon. The intuition behind this idea is that the aggressive investors are more sensitive to transaction costs and thus need to be compensated for these costs in equilibrium. Applying their model to CDS market data, Bongaerts et al (2011) find that sellers of credit protection receive illiquidity compensation on top of the compensation for default risk (liquidity premium). In our research paper, we investigate the effect of illiquidity on CDS premia. The illiquidity variable is proxied by the residuals from a regression of CDS bid-ask spread on equity volatility and past CDS premium. The residuals represent the pure illiquidity component of the CDS bid-ask spread²⁷.

The dispersion of analysts’ forecasts on firm’s earnings over a forecasting period of three months represents the measure of uncertainty of market participants over the fundamentals and the future performance of the firms²⁸. The underlying hypothesis is that greater uncertainty can increase CDS premia above the tail-model predictions when investors are adverse to uncertainty. Moreover, uncertainty (or ambiguity) can induce non-participation in the market. In the Easley and O’Hara (2010)

²⁶The demand-supply imbalance and the aggressiveness of credit protection sellers are fundamental to explain liquidity effects on credit derivatives, besides the “hedging argument”. The price of the CDS insurance in fact can rise also when the underlying market in which the writers of the insurance do their hedging becomes less liquid.

²⁷Following Bongaerts, de Jong and Driessen (2011), the CDS liquidity level on day t is computed as summation of half bid-ask spread on day $t - 1$ plus half bid-ask spread on day t : CDS illiquidity level = $\hat{A} \frac{1}{2}$ CDS BA (t) + $\hat{A} \frac{1}{2}$ CDS BA ($t-1$). A regression of CDS illiquidity level on past CDS premium and contemporaneous equity volatility is performed over a rolling window of 180 days. The residuals represent a “pure illiquidity” component of the CDS bid-ask spread (i.e. orthogonal to past price level and equity volatility), which is employed as final proxy of illiquidity in the CDS market. As alternative illiquidity proxy we employ also equity illiquidity, i.e. the residuals from a regression of Equity illiquidity level = $\hat{A} \frac{1}{2}$ Equity BA (t) + $\hat{A} \frac{1}{2}$ Equity BA ($t-1$) on past equity price and equity volatility (pure illiquidity component of equity bid-ask spread). Marra (2012) shows that the illiquidity in equity and CDS markets tend to co-move across large number of firms, particularly during the crisis periods. Consistently, the results of the regressions of tail-model errors on equity illiquidity and CDS illiquidity tend to be similar in terms of statistical significance. However, the impact of equity illiquidity appears larger in terms of economic significance.

²⁸We proxy the belief disagreement (uncertainty) about future firm’s earnings using the analysts’ earnings forecast dispersion, i.e. the ratio between the median of earnings forecasts for each firm across all analysts and the relative standard deviation. We follow the methodology of Buraschi et al (2010). We use analysts’ forecasts of earnings per share, taken from the Institutional Brokers Estimate System (I/B/E/S) database. This database contains individual analyst’s forecasts organized by: (A) forecast date; and (B) the last date the forecast was revised and confirmed as accurate. Following Buraschi et al (2010), and Diether, Malloy, and Scherbina (2002), we use only stock-split unadjusted data. As initial step, we match analysts’ forecast data with our CDS data. We extend each forecast date to its revision date: if, for example, a forecast is made in January 2006 and it is last confirmed in March 2006, we use this forecast for January, February, and March 2006. If more than one forecast per month is recorded for the same analyst, we use the forecast which was confirmed most recently.

model of investors' behaviour in presence of uncertainty, when traders have incomplete preferences over assets, absence of trading and higher illiquidity can arise. We test whether this theory has any implication for inducing larger spreads in the CDS market for investment-grade firms, particularly over the turbulent period of the crisis. The debate on uncertainty and risk aversion has not been deeply examined in this research paper. However, for this analysis the dispersion of analysts' forecasts on firms' earnings is chosen as the measure of uncertainty because it should not overlap with other measures which proxy both investors' aversion to risk and to uncertainty (e.g. VIX). It is in fact generally difficult to disentangle one from the other.

To capture the illiquidity of the firms in cross-section we also use firms' market capitalization. Securities issued by small firms and derivatives written on these claims tend to be more illiquid than the corresponding claims on large firms. Moreover, the level of uncertainty on small firms' performance tends to be higher given the limited number of analysts following those firms.

Graphical evidence in Figure 9 shows a positive relationship on average between tail-model premia errors and CDS illiquidity over the crisis week (after 21 September 2008). Figure 10 also shows that the tail-model implied volatility errors tend to be positively related to CDS illiquidity. The cross-sectional regression results in Tables 5, 6, 7, and 8 confirm that CDS illiquidity has a significantly positive impact on tail-model errors in the turbulent week (21 September 2008), but no effect in the calmer week (19 March 2006). In the week after the Lehman Brothers failure, a 1 standard deviation change in CDS illiquidity increases tail-model premia errors and implied volatility errors respectively by 0.36 (0.46 if we control for equity volatility) and 0.44 standard deviations (0.57 if we control for equity volatility). Over the same week the variable measuring investors' dispersion of beliefs and uncertainty also becomes highly significant: a 1 standard deviation change in uncertainty increases tail-model premia errors and implied volatility errors respectively by 0.74 (0.37 if we control for equity volatility) and 0.94 standard deviations (0.41 if we control for equity volatility). Graphical evidence in Figures 11 and 12 also shows a positive relationship between the tail-model errors and the measure of uncertainty (although the graphical relationship appear to be driven by a handful of data points, both variables have been preliminary filtered from outliers).

6 Time-Series Fit of the Tail-Model and further Analysis of Determinants of Tail-Model Errors

The time-series analysis of the average CDS premium and implied asset volatility estimated from the tail-model (in-sample) across 40 highly-rated firms shows that the dynamics predicted by the tail-model follow the expected pattern, with a sharp increase of average CDS premium over the crisis period (see Figures 13 and 14). Although the average premium estimated from the Merton model calibrated on Schaefer and Strebulaev (2008) volatility (SS model) seem to capture better some of the explosion of the credit premia at the peak of the crisis (2008), it performs worse than the tail-model in the pre and post-crash period. In particular, before the second quarter of 2008, the SS model - reliant on lower historical volatilities - largely underestimates the average market

CDS premium. The predictions from the tail-model look on average more reasonable, although they remain insufficient to explain completely the observed size of the CDS premia during the crisis period.

We perform time-series regressions to identify whether average CDS market illiquidity and market uncertainty contribute to explain the tail-model under-predictions. We also test for the effect of market volatility. Although this effect has been already captured by the auxiliary panel regression in Equation (4), it might appear again in tail-model prediction errors for the following reasons:

- (1) The measure of market volatility used in the auxiliary panel regression may not capture the whole volatility-effect;
- (2) The model specification (linear) used in the panel regression may describe the relationship between CDS premia and market volatility only at cost of an approximation;
- (3) The CDS sensitivity to equity volatility estimated from the panel regression is a physical measure, but it is then used in the risk neutral domain (for model calibration) to predict CDS premia and implied volatility. However, the risk neutral domain disregards the effect of volatility risk;
- (4) Literature on uncertainty and ambiguity aversion in financial markets points out that higher market volatility is one of the key indicators of increased market uncertainty.

The effects of market volatility on tail model errors are captured using two alternative market volatility proxies: VIX index (Q-measure) and S&P500 index EWMA volatility (P-measure).

Over the crisis period the average tail-model errors appear closely related to: (i) measures of market volatility (VIX and S&P500 volatility - see Figures 15 and 16); (ii) average firms' illiquidity (measured by average residual equity bid-ask spread - see Figure 17); and (iii) median firms' earnings uncertainty (see Figure 18). The time-series regressions in Tables 9 and 10 confirm that higher average CDS illiquidity and VIX contributes to increase the tail-model errors. An increase of one standard deviation in VIX leads on average to 0.62 standard deviations increase in the tail-model premia error. One standard deviation increase in average CDS residual bid-ask spread determines 0.38 standard deviations increase in average tail-model premia error. VIX is not significant to explain the increase in implied volatility error, whereas CDS illiquidity is economically and statistically significant. The proxy for market uncertainty is not found significant in time-series when we control also for VIX and average CDS illiquidity (unreported result).

Further, to examine the influence of illiquidity, uncertainty, and volatility risk we also employ panel analysis on tail-model prediction errors. In the panel regressions tail-model errors for CDS premia and implied volatility are regressed on VIX index, earnings' forecast uncertainty, CDS illiquidity, and inverse of market capitalization. The panel regressions' results reported in Tables 11 and 12 show that all explanatory variables are positively significant to explain the increase in CDS premia tail-model errors. In particular, the tail-model errors increase with higher earnings' forecast uncertainty and CDS illiquidity. The regressions are performed with and without fixed (firm) effects²⁹.

Finally, Table 13 shows the pair-wise Pearson correlations between all frictional variables (and the

²⁹Only the former results are reported for brevity. Excluding fixed firm effects does not change qualitatively and quantitatively the results.

statistical significance of these correlations). The numbers in Table 13 demonstrate that collinearity may be a potential issue only between VIX and market capitalization, which two variables (for this reason) are never included together in the panel regressions (see Tables 11 and 12).

7 Forecasting Ability of the Tail-Model

So far we have been working with data “in sample” to test the validity of the tail-model. We have detected significant influences of illiquidity and uncertainty on the component of CDS premia that the tail-model leaves unexplained. In this Section, by means of out-of-sample forecasting, we perform a more severe test on the predictive power of the tail-model and the illiquidity and uncertainty factors for CDS premia. This test verifies whether: (1) the tail-model and its adjustment to illiquidity and uncertainty can be used to predict future CDS premia correctly; and (2) the tail-model outperforms traditional methodologies of Merton model calibration based on historical volatility estimates (such as the one by Schaefer and Strebulaev, 2008).

For the rolling forecasting exercise we define as the initial in-sample period 1/1/2005 to 6/1/2008 (157 weeks). The out-of-sample period, over which we predict the CDS premia, includes the last two years of the sample and goes from 13/1/2008 to 27/12/2009.

Firstly, we estimate $dCDS/d\sigma^E$ in sample by performing the auxiliary panel regression in Equation (4) on a rolling window of 157 weeks. Secondly, we calibrate the Merton model out of sample to the sensitivity measure $dCDS/d\sigma^E$ estimated in-sample to obtain the tail-model forecasts of CDS premia³⁰. Thirdly, for each week t over the out-of-sample period and each firm i we obtain a forecast of the CDS premium from an *adjusted tail-model* as follows:

Forecast of CDS Premium from Adjusted Tail-Model =

Forecast of CDS Premium from Tail-Model (as defined above) + Adjustment Factor

The adjustment factor is given over the out-of-sample period by:

Estimated Intercept + $Beta^{ILL} \times$ CDS Illiquidity $_t$ + $Beta^{MC} \times$ Inverse Market Capitalization $_t$ + $Beta^{FORDISP} \times$ Earnings’ Forecast Dispersion $_t$.

The intercept and the coefficients $Beta^{ILL}$, $Beta^{MC}$, and $Beta^{FORDISP}$ are estimated in-sample from a rolling panel regression of tail-model premia errors (previously obtained using the whole sample of data, as explained in Section 4) on CDS residual bid-ask spread, inverse market capitalization, and earnings’ forecast dispersion³¹. Using the adjustment factor we can test whether taking into account the *past* influence of illiquidity and uncertainty on CDS premia tail-model errors can improve CDS premia predictions *for the future* (up to two years ahead), above the tail-model predictions.

³⁰In the rolling forecasting exercise the CDS premium over the week commencing on 13/1/2008 is obtained using the $dCDS/d\sigma^E$ estimated from the panel regression over the sample 1/1/2005 - 6/1/2008. Next, the CDS premium for the week commencing on 20/1/2008 is obtained using the $dCDS/d\sigma^E$ estimated from the panel regression over the sample 8/1/2005 - 13/1/2008 (i.e. the initial week of the previous in-sample period is dropped, while one “new week” is added at the end; the length of the in-sample period remains however fixed to 157 weeks).

³¹The prediction of the adjustment factor has been repeated using only CDS residual illiquidity and analysts’ earnings forecast dispersion (i.e. without including the market capitalization variable). Results remain mostly unchanged, therefore are not reported for brevity.

The average results from the forecasting exercise are illustrated in Figure 19. In this graph the adjusted tail-model forecasts are compared with the market CDS premia, the forecasts obtained using the tail-model (with no adjustments), and the forecasts from the SS model. Across all 40 investment-grade firms, the adjusted tail-model performs on average very well in forecasting the level of the CDS market premia out of sample. The SS model has lower forecasting ability. On average, the forecasts obtained by the SS model systematically under-predict the market CDS premia over the whole period before November 2008 and again starting from April 2009. The forecasting ability of the tail-model using $dCDS/d\sigma^E$ is good both in magnitude and direction. The summary statistics for the absolute forecast errors (Mean Squared Forecast Errors - MSE, Root Mean Squared Forecast Errors - RMSE, and Mean Absolute Forecast Errors - MAE) reported in Table 15 confirm the relative forecasting performances: the tail-model and its version adjusted to illiquidity and uncertainty over-perform the traditional SS methodology in predicting CDS premia out of sample. The RMSE and MAE of the SS methodology are three times the RMSE and MAE of the adjusted tail-model which delivers the best forecasts.

Finally, as a robustness check, the rolling forecasting exercise is repeated on different in and out-of-sample periods, respectively 1/1/2005 to 3/25/2007 and 1/4/2007 to 27/12/2009. The starting in-sample period coincides with the pre-crisis period. The results are shown in Figure 20 and Table 16, and confirm the good performance of the tail-model, and the even better performance of the adjusted tail-model.

8 Robustness Check: Explicit Model vs. Reduced-Form Model

As a further robustness check on the tail-model, in this Section we firstly investigate whether the errors of the tail-model are related to the errors of the initial (auxiliary) panel regression in Equation (4), where market CDS premia are regressed on interest rate, leverage ratio, and equity volatility variables, conditional on VIX levels (Table 4 reports the results of the relative panel estimation). In this Section we call the tail-model also the “explicit structural model” and the auxiliary panel regression in Equation (4) the “reduced-form structural model”.

If we find that the tail-model errors are larger than the panel errors and unrelated to them, this means that the Merton model calibrated on $dCDS/d\sigma^E$ introduces further sources of disturbance. If we find that the errors are similar, then it is likely that both the panel regression and the tail-model omit to consider the same variables. Our hypothesis is that the omitted variables are CDS market illiquidity and investors’ aversion to uncertainty. After estimating the auxiliary panel regression in Equation (4), we construct the residual series (i.e. the panel errors) and compare them with the errors from the tail-model. Figure 21 shows that the tail-model errors are strongly and positively related to the panel errors.

Next, we include illiquidity and uncertainty frictions in the auxiliary panel regression to test whether they are significant to explain market CDS premia; likewise they have been found to explain tail-model errors in the analysis in Sections 5 and 6. This result is displayed in Table 14. Illiquidity and uncertainty are highly significant variables to explain an increase in CDS premia. This confirms their role as possible relevant omitted variables in both the tail-model (explicit model) and the panel regression (reduced-form model).

Tables 4 and 14 show that the *reduced-form model* for CDS premia, represented by the panel regression in Equation (4) and its enhanced version including illiquidity and uncertainty frictions, are able to explain over 57% of the variation in CDS premia. Looking at the high R-squared from the reduced-form model estimation, one might therefore argue that calibrating the explicit tail-model is unnecessary to obtain good CDS premia predictions. However, the explicit tail-model is calibrated on a measure of left-skewness in the firm's returns distribution ($dCDS/d\sigma^E$) which is not incorporated in the panel regression and which has been shown essential to capture cross-sectional and time-series dynamics in credit spreads. To support this argument, we compare the out-of-sample predictive ability of the explicit model with the predictive ability of the reduced-form model (with and without illiquidity and uncertainty variables). The results are presented in Figure 22 and Table 15 (for out-of-sample period January 2008 - December 2009) and Figure 23 and Table 16 (for the longer out-of-sample period April 2007 - December 2009). Table 15 illustrates the measures of forecasting performance for the SS model, the tail-model, the adjusted tail-model, the panel regression, and its adjusted version including illiquidity and uncertainty variables. The forecasts of CDS premia obtained from the adjusted panel regression have on average twice as large mean absolute forecast errors as the forecasts obtained from the adjusted tail-model and more than double root mean squared forecast errors. The forecasting ability of the adjusted tail-model is higher than the one of the tail-model; however, the tail-model forecasts outperform the panel forecasts by 8% (for MAE) and 19% (for RMSE). In turn, the panel regressions can forecast CDS premia out-of-sample better than the Merton model calibrated on SS volatility. Table 16 show that when a longer out-of-sample period is considered (April 2007 - December 2009), the forecasting performance of the reduced-form structural model improves over the tail-model. The MAE and RMSE of the panel forecasts are almost the same as the MAE and RMSE of the tail-model; however, the measures of dispersion of the panel forecast errors are also almost twice as large. Noticeably, in all cases the tail-model adjusted to illiquidity and uncertainty remains the "unbeaten" methodology to forecast CDS premia out-of-sample.

To sum up, although the reduced-form model can forecast average CDS premia quite correctly, the tail-model adjusted for illiquidity and uncertainty returns the highest performance and the closest predictions (it has in fact smaller RMSE and MAE).

9 Conclusions

Previous literature has found that the structural Merton (1974) model is successful in estimating changes in firms' credit risk exposure (Schaefer and Strebulaev, 2008 - SS), but unsuccessful in reproducing the level of credit spreads observed in the market (Eom, Helwege and Huang, 2004; Collin-Dufresne, Goldstein and Martin, 2001; and Huang and Huang, 2004). This has generated the so-called *credit spread puzzle*. Despite its flaws, the Merton (1974) model offers an intuitive economic interpretation of default events and views firms' equity and credit claims as contingent claims written on firms' underlying assets. The structural model literature has developed therefore in parallel to the literature on Black-Scholes for equity options.

In this paper we use a Merton model (with target leverage at its local level), but we calibrate it to a measure of the sensitivity of credit default swap premia to equity volatility $dCDS/d\sigma^E$, rather than to more direct proxies for asset volatility. We call this calibration methodology the "tail-model". The benefit of the tail-model is to capture the variation in the underlying risk-neutral distribution (RND) of firm's asset, not only in the volatility dimension, but also in higher order moments (i.e. skewness and kurtosis). The measure helps to demonstrate that there is a fat tail in the RND and conveys a larger set of information on the time-varying attitude of investors towards the probability of extreme crashes. The model does not however investigate explanations for the existence of the fat tail, which could be due to jumps, correlation risk, volatility risk, and/or investors' risk aversion.

Consistent with the literature on put options, we observe that the implied volatilities from CDS premia for 40 U.S. highly-rated companies (obtained by inverting the Merton model pricing equation) demonstrate the existence of a volatility smirk (in leverage). The $dCDS/d\sigma^E$ calibration can generate the same kind of smirk, which more traditional calibration approaches (such as SS) cannot. In addition, we find that the tail-model largely improves the in-sample and out-of-sample predictions of credit premia relative to more traditional calibration methodologies, before and during the recent crisis period. In particular, the Schaefer and Strebulaev (2008) model works very badly in cross-section. The tail-model performs better than the SS model in both cross-section and time-series.

Nevertheless, the calibration of the tail-model returns CDS premia which are still below those observed in the market. This leads to a consideration of non-default components of credit spreads. In particular, this work carries out a test on the effects of illiquidity and uncertainty on CDS premia. We find that illiquidity in credit markets and investors' aversion to uncertainty on the fundamental values and earnings' prospects of the firms influence investors' attitude and trading decisions in the credit markets, and have a positive impact on residual CDS premia. The tail-model adjusted to include also illiquidity and uncertainty premia replicates the patterns of market CDS premia very closely and displays the lowest average prediction errors in two separate out-of-sample forecasting exercises, when compared to more traditional calibration methodologies.

To the best of our knowledge, no previous empirical work has jointly evaluated the impact of changes in the shape of the risk neutral distribution (and investors' preferences) and illiquidity and uncertainty

frictions on the spreads of CDSs written on investment-grade firms to explain the credit spread puzzle. This work proposes solutions to overcome the difficulty in capturing investors' preferences, their appreciation of the probability of extreme losses, the cost of their inactive participation into the market, and their aversion to uncertainty; and shows that all these factors have a prominent effect on credit premia of highly-rated firms, in particular during the 2007-09 crisis.

References

- AVRAMOV, D., G. JOSTOVA, AND A. PHILIPPOV (2007): “Understanding Changes in Corporate Credit Spreads,” *Financial Analysts Journal*, 63(2), 90–105.
- BENKERT, C. (2004): “Explaining Credit Default Swap Premia,” *Journal of Future Markets*, 24(1), 71–92.
- BHARATH, S. T., AND T. SHUMWAY (2008): “Forecasting Default with the Merton Distance to Default Model,” *Review of Financial Studies*, 21(3), 1339–1369.
- BLACK, F., AND M. SCHOLES (1973): “The Pricing of Options and Corporate Liabilities,” *Journal of Political Economy*, 81(3), 637–654.
- BURASCHI, A., F. TROJANI, AND A. VEDOLIN (2011): “Economic Uncertainty, Disagreement, and Credit Markets,” Working Paper.
- CAMPBELL, J. Y., AND G. B. TAKSLER (2003): “Equity Volatility and Corporate Bond Yields,” *Journal of Finance*, 58(6), 2321–2350.
- CHEN, L., D. A. LESMOND, AND J. WEI (2007): “Corporate Yields Spreads and Bond Liquidity,” *Journal of Finance*, 62(1), 119–149.
- COLLIN-DUFRESNE, P., AND R. S. GOLDSTEIN (2001): “Do Credit Spreads Reflect Stationary Leverage Ratios?,” *Journal of Finance*, 56(5), 1929–1957.
- COLLIN-DUFRESNE, P., R. S. GOLDSTEIN, AND Y. F. (Forthcoming, 2012): “On the Relative Pricing of Long Maturity Index Options and CDX Tranches,” *Journal of Finance*.
- COLLIN-DUFRESNE, P., R. S. GOLDSTEIN, AND J. HELWEGE (2003): “Is Credit Event Risk priced? Modeling Contagion via the Updating of Beliefs,” Working Paper Carnegie Mellon University.
- COVAL, J. D., J. W. JUREK, AND E. STAFFORD (2009): “The Pricing of Investment Grade Credit Risk during the Financial Crisis,” Harvard University Working Paper.
- CREMERS, K. J. M., J. DRIESSEN, AND P. MAENHOUT (2008): “Explaining the Level of Credit Spreads: Option-Implied Jump Risk Premia in a Firm Value Model,” *Review of Financial Studies*, 21(5), 2209–2242.
- CREMERS, K. J. M., J. DRIESSEN, P. MAENHOUT, AND D. WEINBAUM (2008): “Individual Stock-Option Prices and Credit Spreads,” *Journal of Banking and Finance*, 32(12), 2706–2715.
- CROSBIE, P., AND J. BOHN (2003): “Modelling Default Risk-Modelling Methodology,” *Moody’s KMV Paper*.
- DELIANEDIS, G., AND R. GESKE (2001): “The Components of Corporate Credit Spreads: Default, Recovery, Tax, Jumps, Liquidity and Market Factors,” UCLA Working Paper.
- DIETHER, K., C. J. MALLOY, AND A. SCHERBINA (2002): “Differences of Opinion and the Cross Section of Stock Returns,” *Journal of Finance*, 57, 2113–2141.
- DUFFIE, D., AND N. GÂRLEANU (2001): “Risk and Valuation of Collateralized Debt Obligations,” *Financial Analysts Journal*, 57(1), 1765–1799.

- EASLEY, D., AND M. O'HARA (2010): "Liquidity and Valuation in an Uncertain World," *Journal of Financial Economics*, 97(1), 1–11.
- EOM, Y. H., J. HELWEGE, AND J. HUANG (2004): "Structural Models of Corporate Pricing: An Empirical Analysis," *Review of Financial Studies*, 17(2), 499–544.
- GEMMILL, G., AND A. KESWANI (2011): "Downside Risk and the Size of Credit Spreads," *Journal of Banking and Finance*, 35(8), 2021–2036.
- GEMMILL, G., AND Y. YANG (2010): "The Equity-Smile and Credit-Spread Puzzles: Are They One and the Same?," Working Paper.
- JACKWERTH, J. C., AND M. RUBINSTEIN (1996): "Recovering Probability Distributions from Option Prices," *Journal of Finance*, 51(5), 1611–1632.
- JARROW, R., AND F. YU (1997): "Counterparty Risk and the Pricing of Defaultable Securities," *Journal of Finance*, 56(5), 1765–1799.
- JONES, E., S. MASON, AND E. ROSENFELD (1984): "Contingent Claim Analysis of Corporate Capital Structure: An Empirical Investigation," *Journal of Finance*, 39, 611–627.
- JORION, P., AND G. ZHANG (2007): "Good and Bad Credit Contagion: Evidence from Credit Default Swaps," *Journal of Financial Economics*, 84(3), 860–883.
- LONGSTAFF, F., AND A. RAJAN (2008): "An Empirical Analysis of the Pricing of Collateralized Debt Obligations," *Journal of Finance*, 63(2), 529–563.
- MERTON, R. (1974): "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449–470.
- SCHAEFER, S. M., AND I. A. STREBULAIEV (2008): "Structural Models of Credit Risk are useful: Evidence from Hedge Ratios on Corporate Bonds," *Journal of Financial Economics*, 90, 1–19.
- VASSALOU, M., AND Y. XING (2004): "Default Risk in Equity Returns," *Journal of Finance*, 59(2), 831–868.
- ZHANG, B. Y., H. ZHOU, AND H. ZHU (2009): "Explaining Credit Default Swap Spreads with the Equity Volatility and Jump Risks of Individual Firms," *Review of Financial Studies*, 22(12), 5099–5131.
- ZHOU, C. (2001): "The Term Structure of Credit Spreads with Jump Risk," *Journal of Banking and Finance*, 25, 2015–2040.

A Figures and Tables

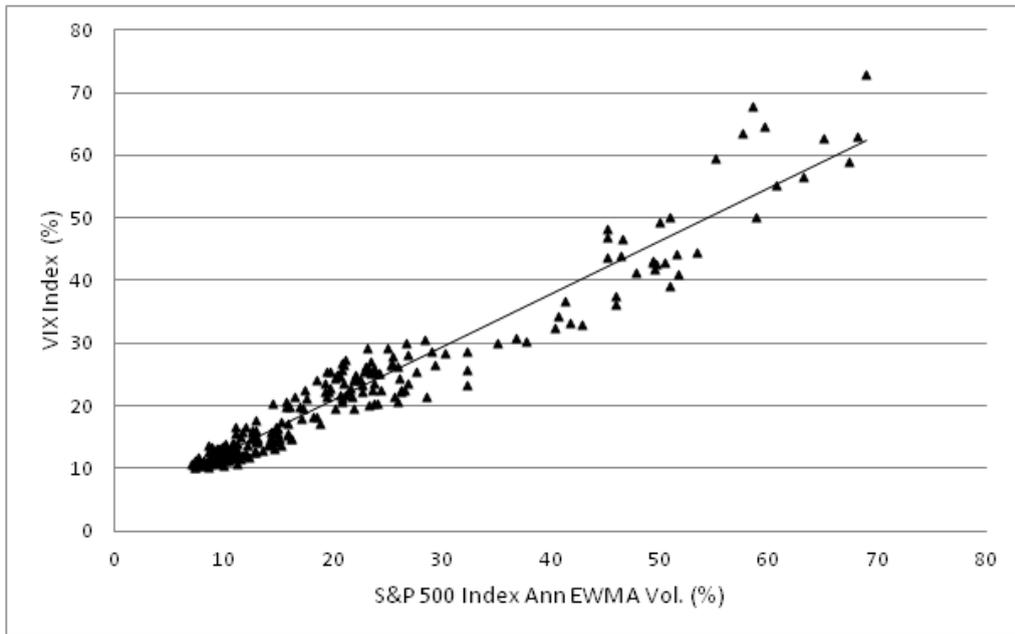


Figure 1: Relationship between VIX Index and S&P 500 Index Annualized Exponentially-Weighted Moving Average Volatility (with $\lambda=0.94$)
(Volatilities are measured in percentage units; Sample: January 2005 - December 2009.)

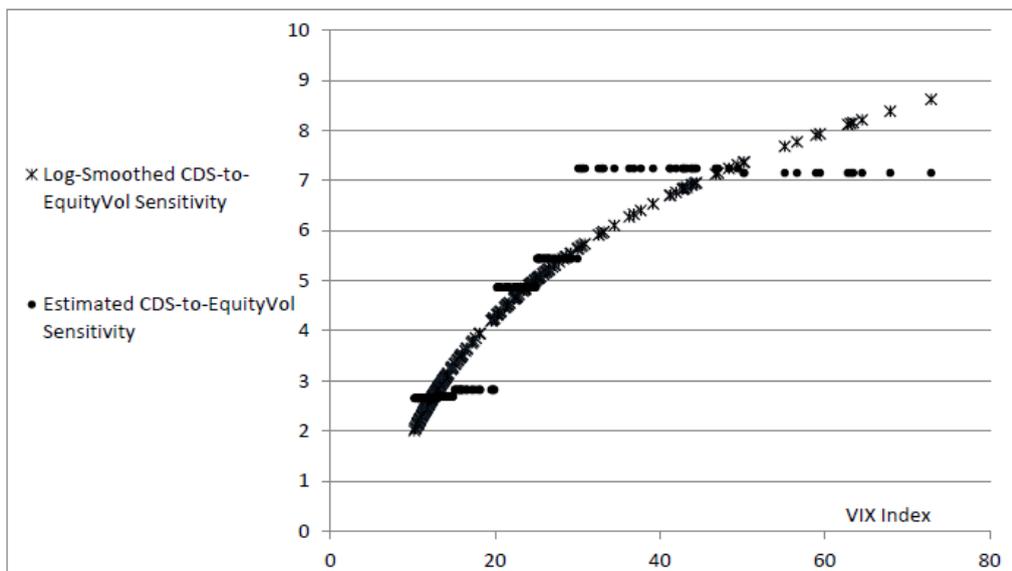


Figure 2: Relationship between VIX Index and Estimated or Logarithm-fitted Sensitivity of CDS Premia to Equity Volatility $dCDS/d\sigma^E$
(All variables are measured in percentage units; Sample: January 2005 - December 2009; 40 U.S. investment-grade firms.)

Figure 3: Cross-section of Implied Volatilities from Market CDS Premia and Implied Volatilities from Tail-Model, against Leverage Ratio
 (19 Mar 2006; 40 U.S. investment-grade firms; Logarithmic curves are fitted to the data; All variables are measured in decimals.)

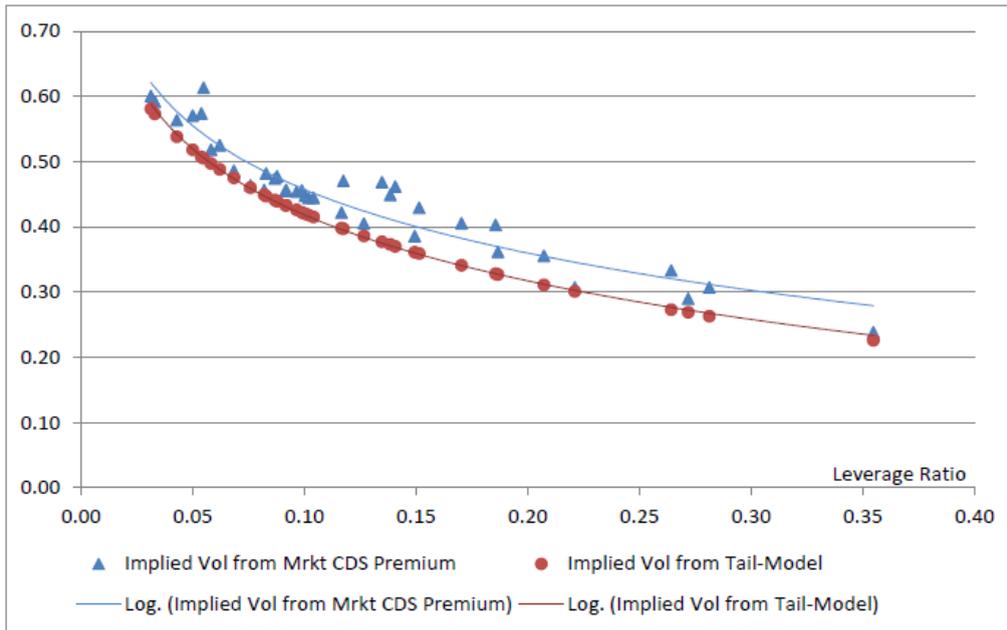


Figure 4: Cross-section of Implied Volatilities from Market CDS Premia, Volatilities estimated from SS (2008) Methodology, and Volatilities estimated using $(1 - \text{Leverage ratio}) \times \text{Annualized EWMA Equity Volatility}$, against Leverage Ratio
 (19 Mar 2006; 40 U.S. investment-grade firms; Logarithmic curves are fitted to the data; All variables are measured in decimals.)

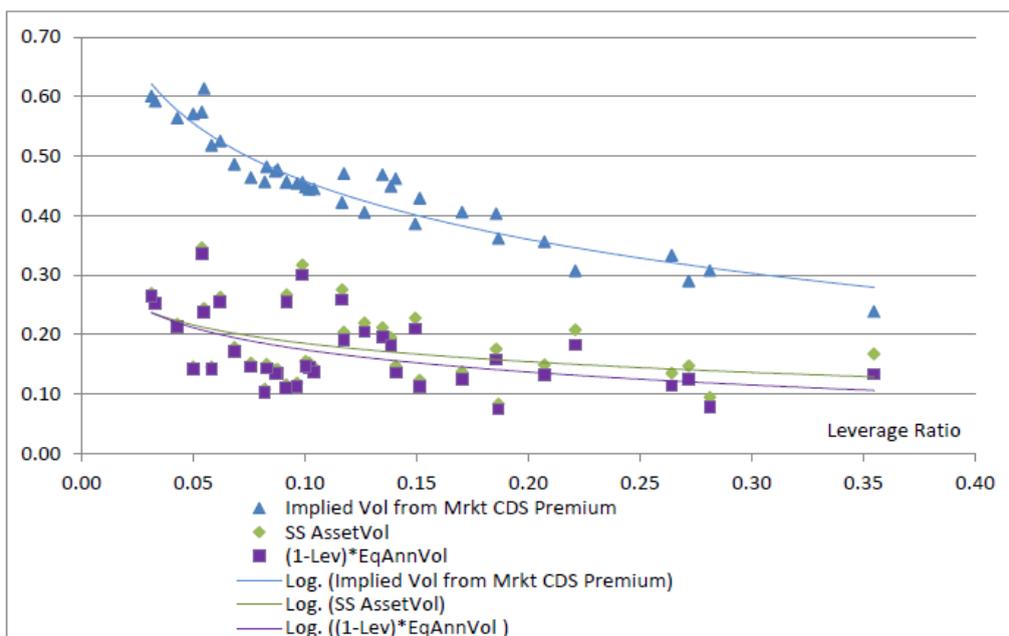


Figure 5: Cross-section of Market CDS Premia, CDS premia implied by Tail-Model using $dCDS/d\sigma^E$, and CDS Premia implied by Merton Model calibrated on SS Volatility, against Leverage Ratio
 (19 Mar 2006; 40 U.S. investment-grade firms; CDS premia are measured in basis points, leverage ratios in decimals.)

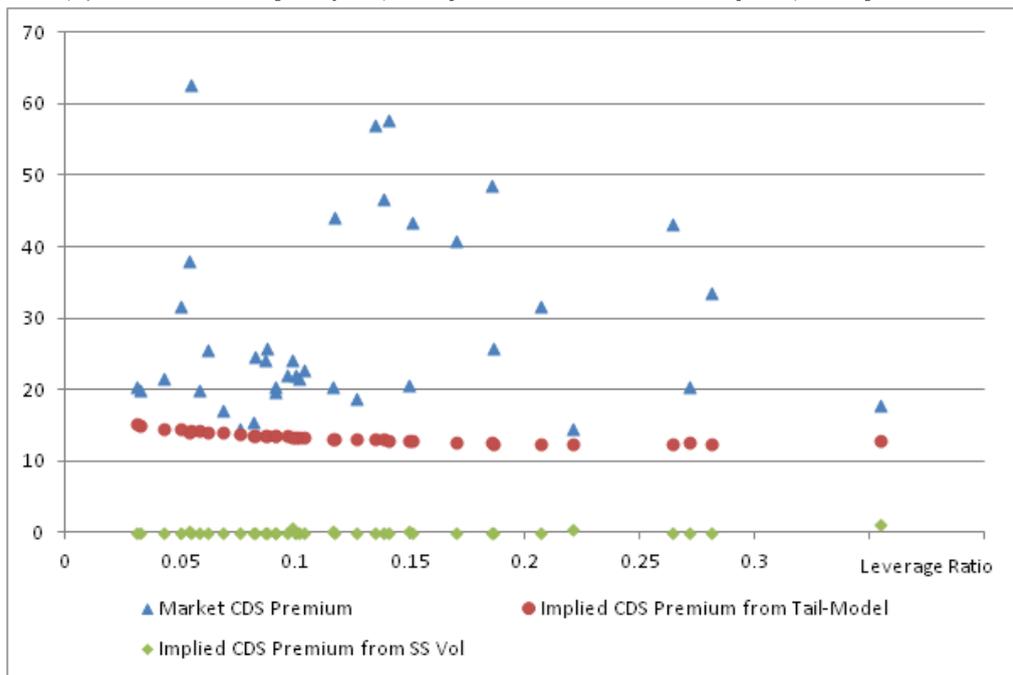


Figure 6: Cross-section of Implied Volatilities from Market CDS Premia and Implied Volatilities from Tail-Model, against Leverage Ratio (21 Sept 2008; 40 U.S. investment-grade firms; Logarithmic curves are fitted to the data; All variables are measured in decimals.)

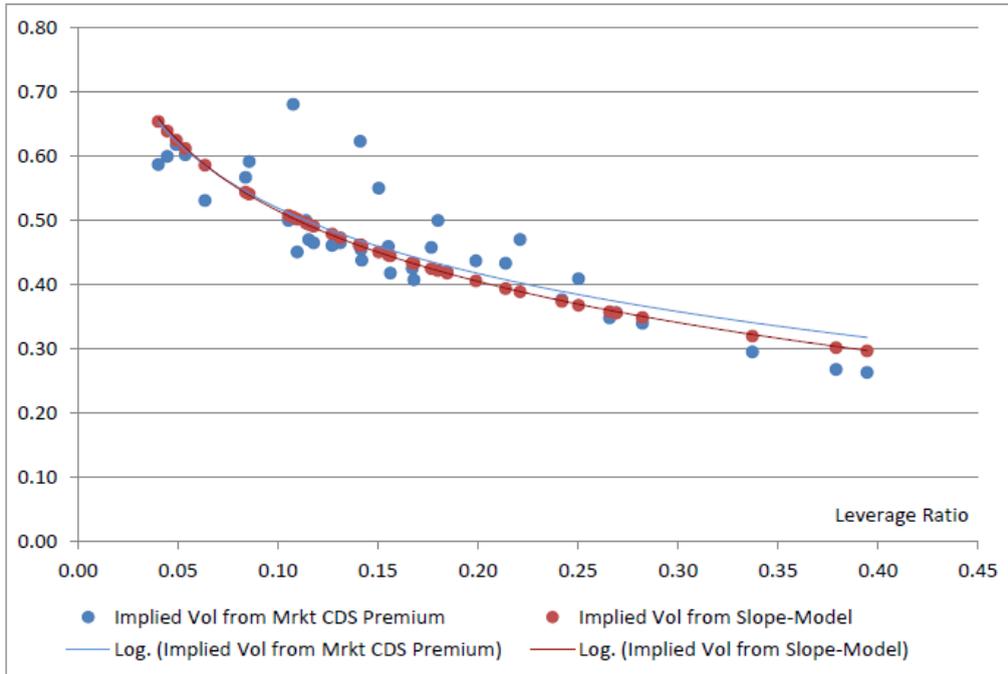


Figure 7: Cross-section of Implied Volatilities from Market CDS Premia, Volatilities estimated from SS (2008) Methodology, and Volatilities estimated using $(1 - \text{Leverage ratio}) \times \text{Annualized EWMA Equity Volatility}$, against Leverage Ratio (21 Sept 2008; 40 U.S. investment-grade firms; Logarithmic curves are fitted to the data; All variables are measured in decimals.)

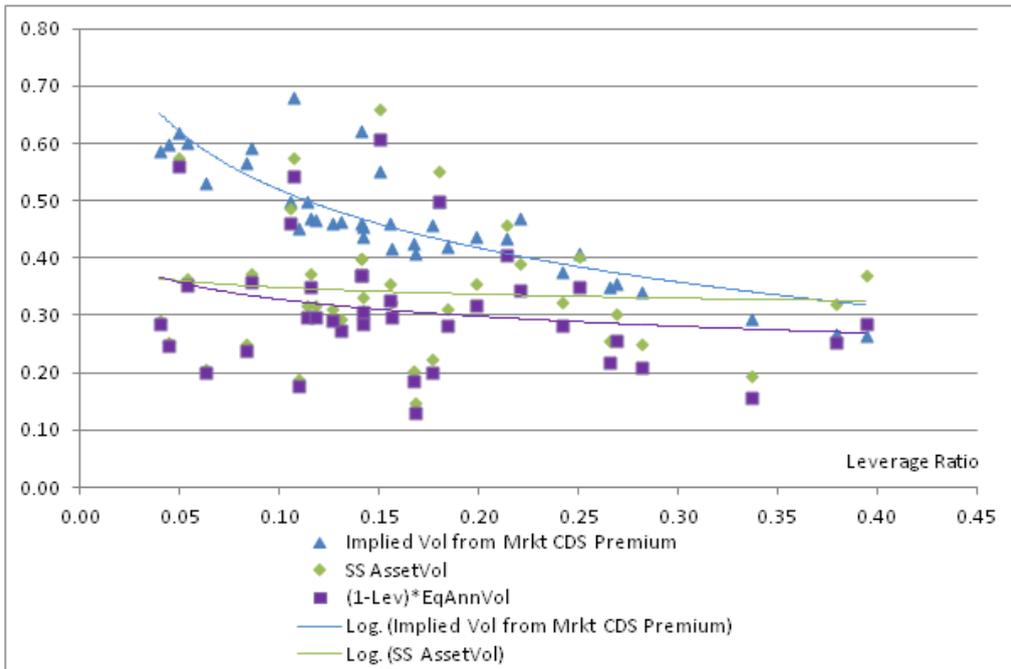


Figure 8: Cross-section of Market CDS Premia, CDS Premia implied by Tail-Model using $dCDS/d\sigma^E$, and CDS Premia implied by Merton Model calibrated on SS Volatility, against Leverage Ratio
 (21 Sept 2008; 40 U.S. investment-grade firms; CDS premia are measured in basis points, leverage ratios in decimals.)

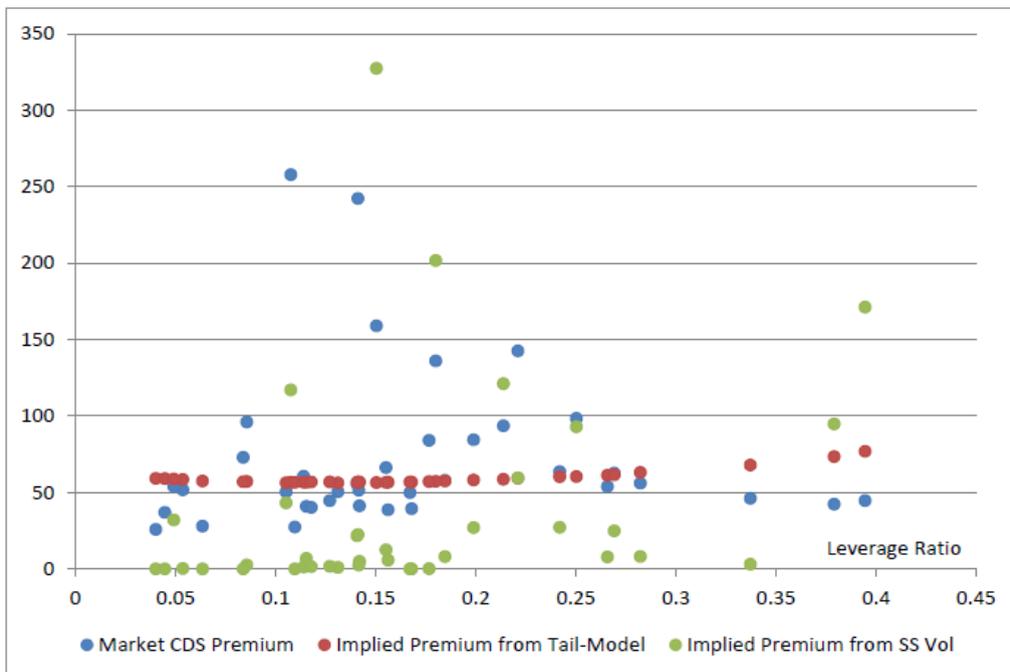


Figure 9: Scatter Plot of CDS Premia Errors from Tail-Model against CDS Illiquidity
*(Tail Model CDS Premium Error = CDS Market Premium - CDS Premium implied by Tail-Model;
 CDS Illiquidity = Residuals from a regression of CDS bid-ask spread on past CDS premium
 and contemporaneous equity volatility; 21 Sept 2008; 40 U.S. investment-grade firms;
 Regression line fitted to the data; Tail-Model CDS Premium Errors are measured in basis points.)*

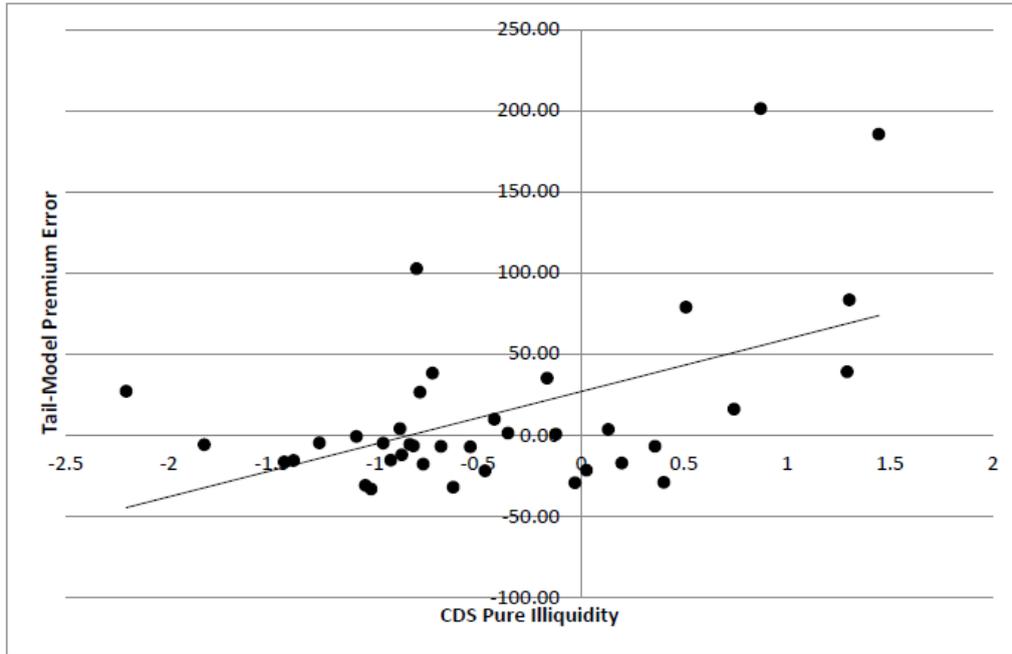


Figure 10: Scatter-Plot of Implied Volatility Errors from Tail-Model against CDS Illiquidity
*(Tail-Model IVol Error = Implied Volatility from CDS Market Premium - Implied Volatility from Tail-Model;
 CDS Illiquidity = Residuals from a regression of CDS Bid-Ask Spread on Past CDS Premium
 and Contemporaneous Equity Volatility; 21 Sept 2008; 40 U.S. investment-grade firms;
 Regression line fitted to the data; Tail-Model CDS Implied Volatility Errors are measured in decimals.)*

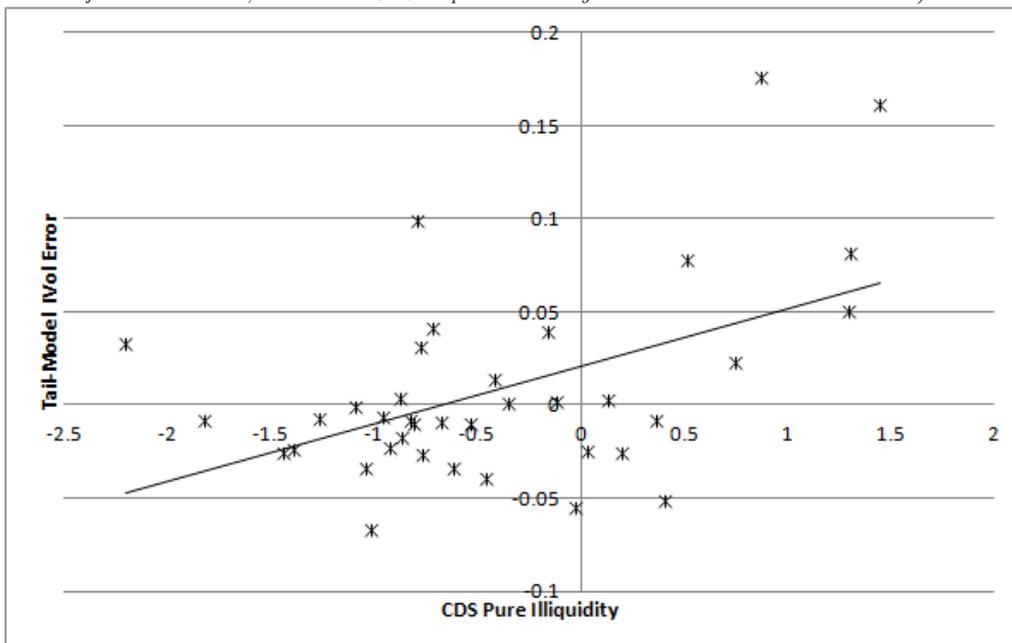


Figure 11: Scatter-Plot of CDS Premia Errors from Tail-Model against Analysts' Earnings Forecast Dispersion (Uncertainty)

(Tail Model CDS Premium Error = CDS Market Premium - CDS Premium implied by Tail-Model;
 Earnings' Uncertainty = Dispersion of Analysts' forecasts on the firm's earnings;
 21 Sept 2008; 40 U.S. investment-grade firms; Regression line fitted to the data;
 Tail-Model CDS Premium Errors are measured in basis points.)

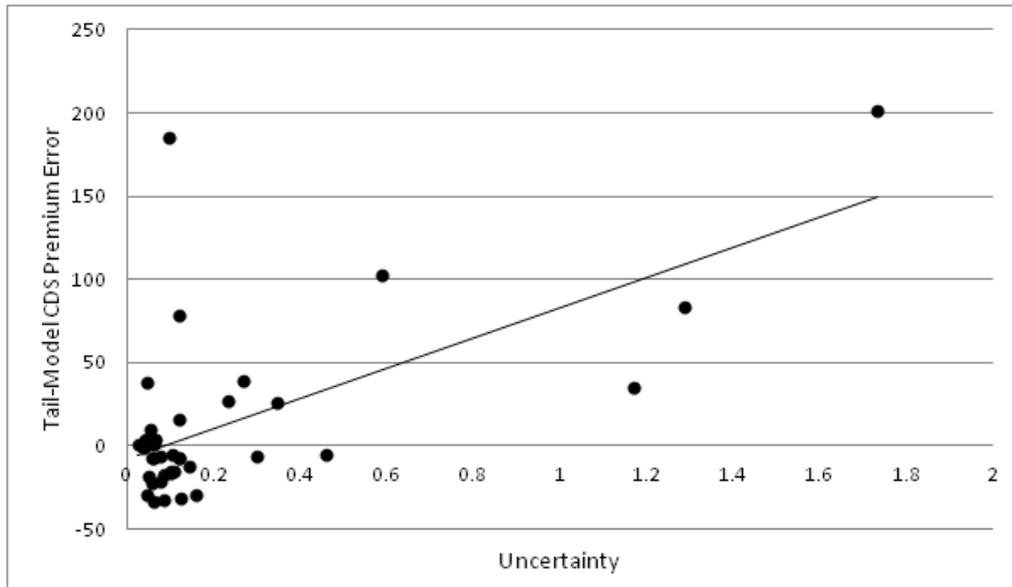


Figure 12: Scatter-Plot of Implied Volatility Errors from Tail-Model against Analysts' Earnings Forecast Dispersion (Uncertainty)

(Tail-Model IVol Error = Implied Volatility from CDS Market Premium - Implied Volatility from Tail-Model;
 Earnings' Uncertainty = Dispersion of Analysts' forecasts on the firm's earnings;
 21 Sept 2008; 40 U.S. investment-grade firms; Regression line fitted to the data;
 Tail-Model CDS Implied Volatility Errors are measured in decimals.)

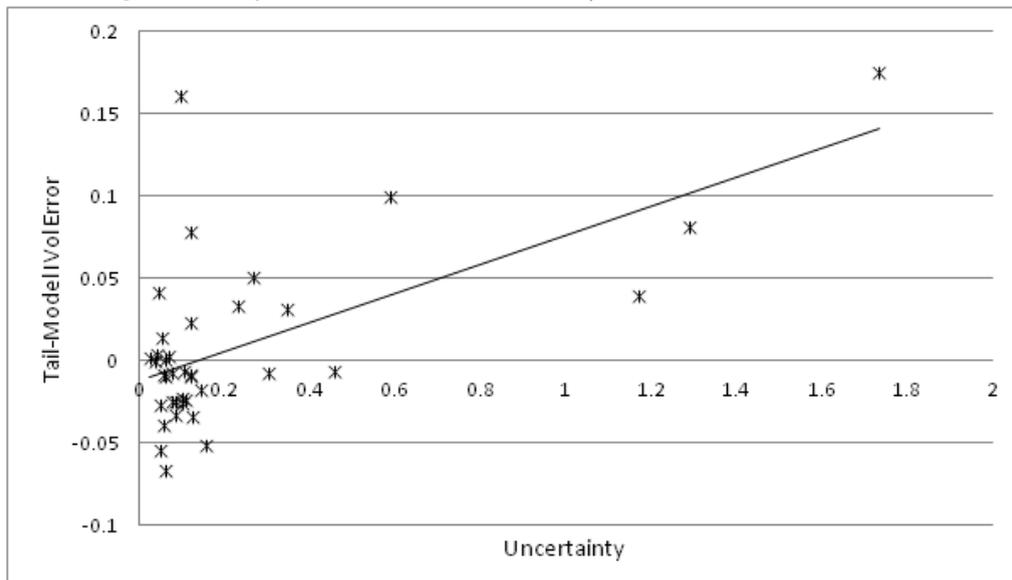


Figure 13: Time-Series of Average CDS Market Premium, Average CDS Premium implied by the Tail-Model using $dCDS/d\sigma^E$, and Average CDS Premium implied by Merton Model calibrated on SS Volatility (40 U.S. investment-grade firms; Sample: Jan 2005 - Dec 2009; Average premia are measured in basis points.)

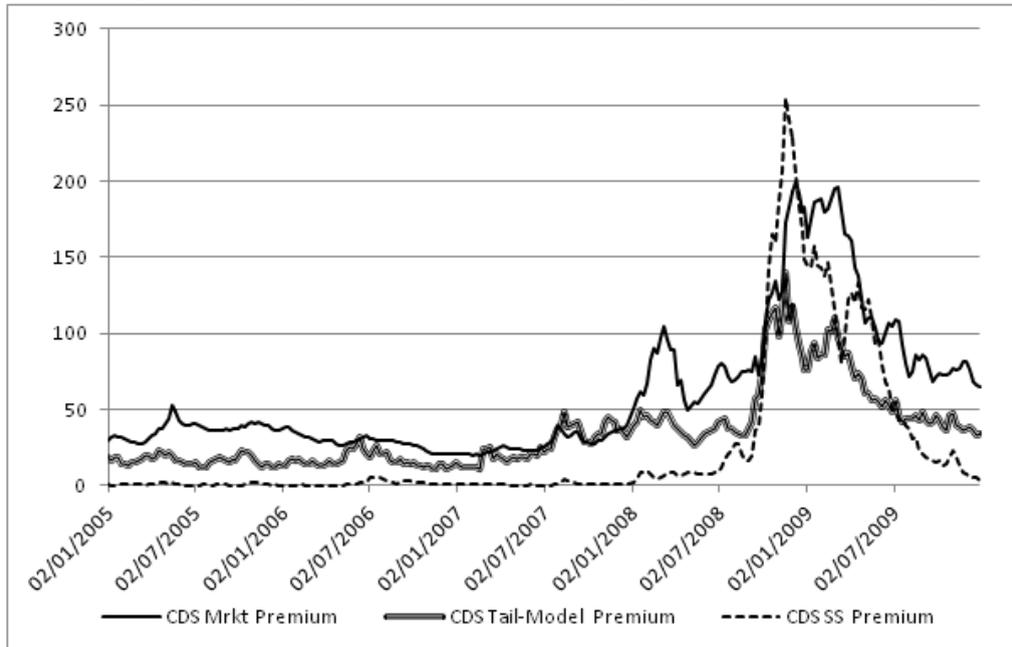


Figure 14: Time-Series of Average Implied Volatility from Market CDS Premia, Average Implied Volatility from Tail-Model using $dCDS/d\sigma^E$, and Average Schaefer-Strebulaev (2008) Volatility (40 U.S. investment-grade firms; Sample: Jan 2005 - Dec 2009; Average volatilities are measured in decimals.)

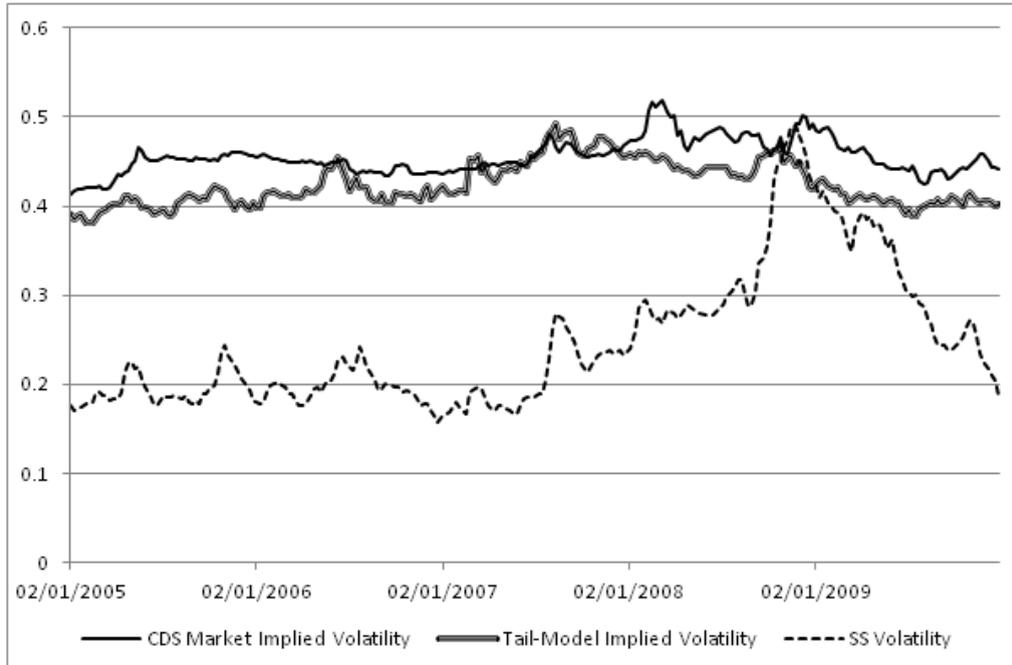


Figure 15: Time-series of Average CDS Premium Error from Tail-Model and VIX Index
(Average Tail-Model CDS Premium Error = Average CDS Market Premium - Average CDS Premium implied by Tail-Model; 40 U.S. investment-grade firms; Sample: Jan 2005 - Dec 2009; Average CDS premium is measured in basis points, VIX in percentage units.)

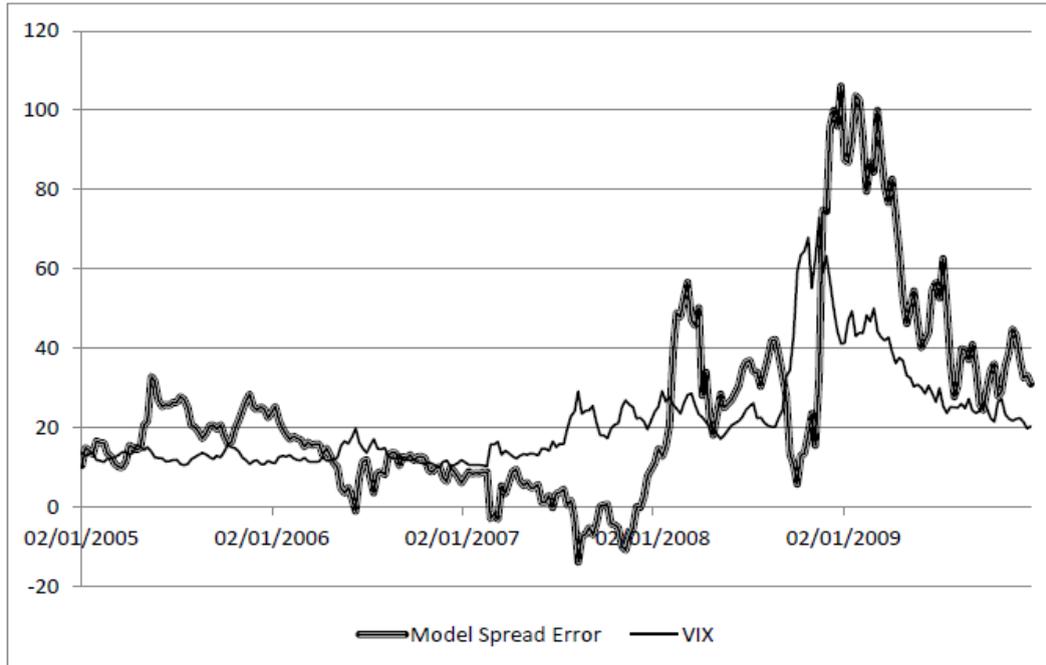


Figure 16: Time-series of Average CDS Premium Error from Tail-Model and Annualized Exponentially-Weighted Moving Average Volatility of S&P500 Index (with lambda=0.94)
(Average Tail-Model CDS Premium Error = Average CDS Market Premium - Average CDS Premium implied by Tail-Model; 40 U.S. investment-grade firms; Sample: Jan 2005 - Dec 2009; Average CDS premium is measured in basis points, S&P500 volatility in percentage units.)

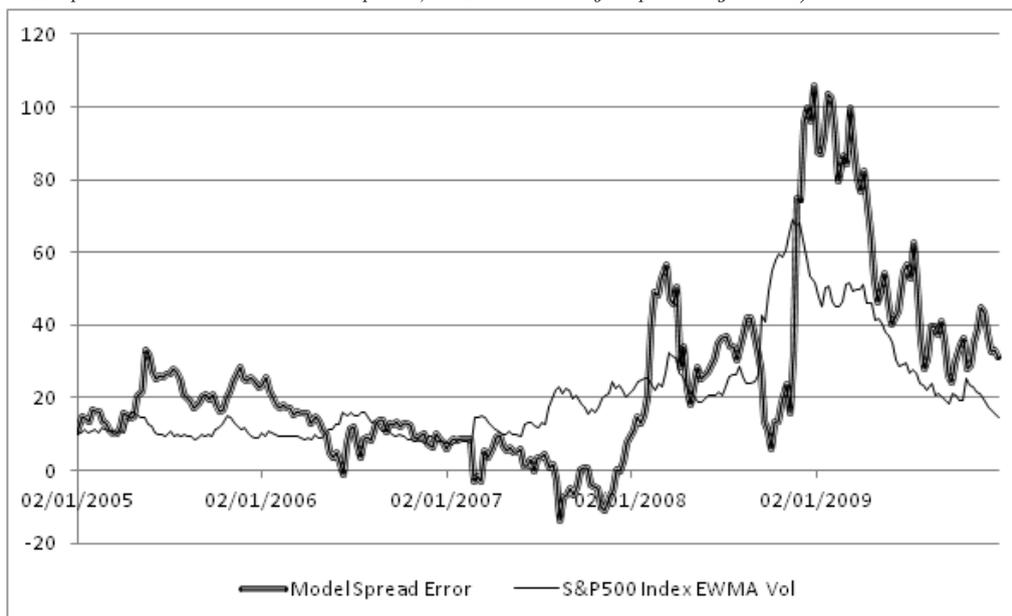


Figure 17: Time-series of Average CDS Premium Error from Tail-Model and Average Equity Residual Illiquidity

(Average Tail-Model CDS Premium Error =
Average CDS Market Premium - Average CDS Premium implied by Tail-Model;
Average Equity Residual Illiquidity =
Average residuals from firm-by-firm regressions of equity bid-ask spreads on equity volatility and past equity prices;
40 U.S. investment-grade firms; Sample: Jan 2005 - Dec 2009;
Average CDS premium and average equity residual illiquidity are measured in basis points.)

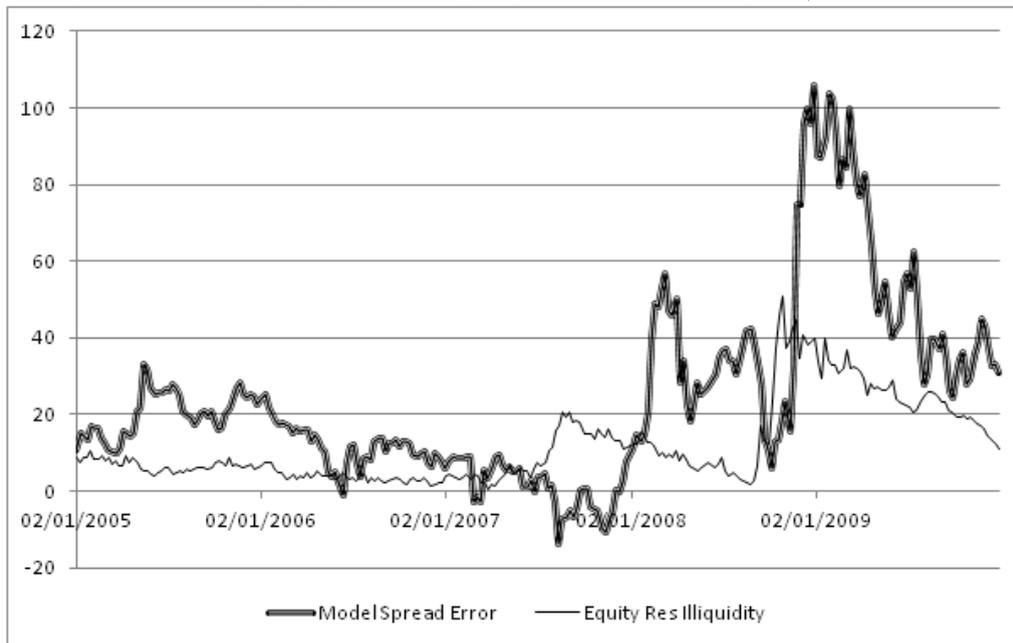


Figure 18: Time-series of Average CDS Premium Error from Tail-Model and Median Analysts' Earnings Forecast Dispersion (Uncertainty) Across Firms

(Average Tail-Model CDS Premium Error =
 Average CDS Market Premium - Average CDS Premium implied by Tail-Model;
 Median Earnings' Uncertainty = Median dispersion of analysts' forecasts on the firm's earnings;
 40 U.S. investment-grade firms; Sample: Jan 2005 - Dec 2009;
 Average CDS premium is measured in basis points, median earnings' uncertainty in percentage units.)

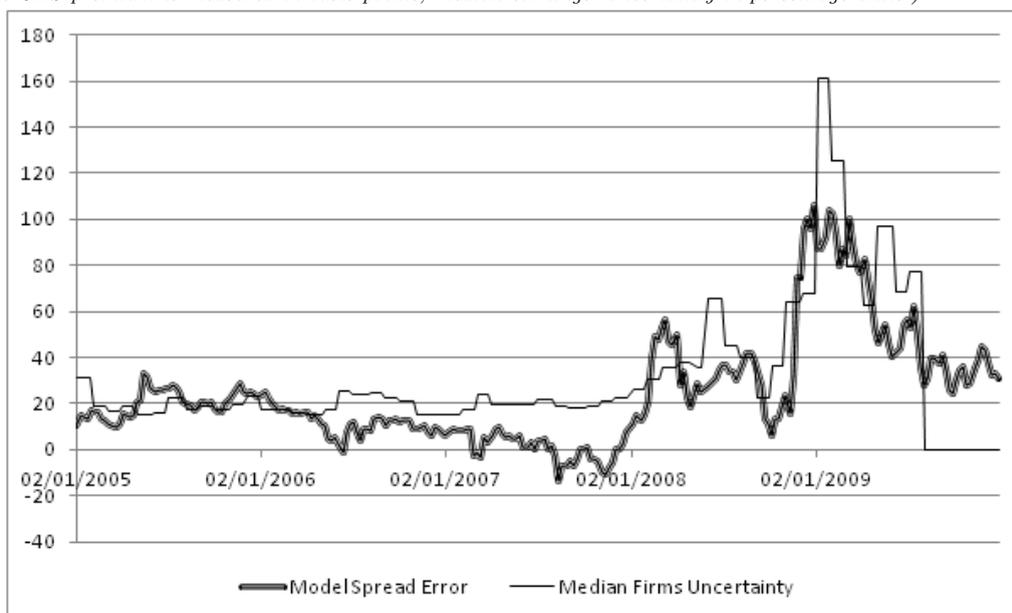


Figure 19: Comparison of Average Out-of-Sample Forecasts:
 CDS Market Premium, CDS Premium from Merton Model calibrated on SS volatility,
 CDS Premium from Tail-Model using $dCDS/d\sigma^E$,
 and CDS premium from Tail-Model adjusted for Illiquidity and Uncertainty
*(Average values across 40 U.S. investment-grade firms; Rolling forecasts;
 In-sample period: 157 weeks starting with 1 Jan 2005 - 6 Jan 2008;
 Out-of-sample period: 13 January 2008 - 31 December 2009;
 Forecasted CDS premia are measured in basis points.)*

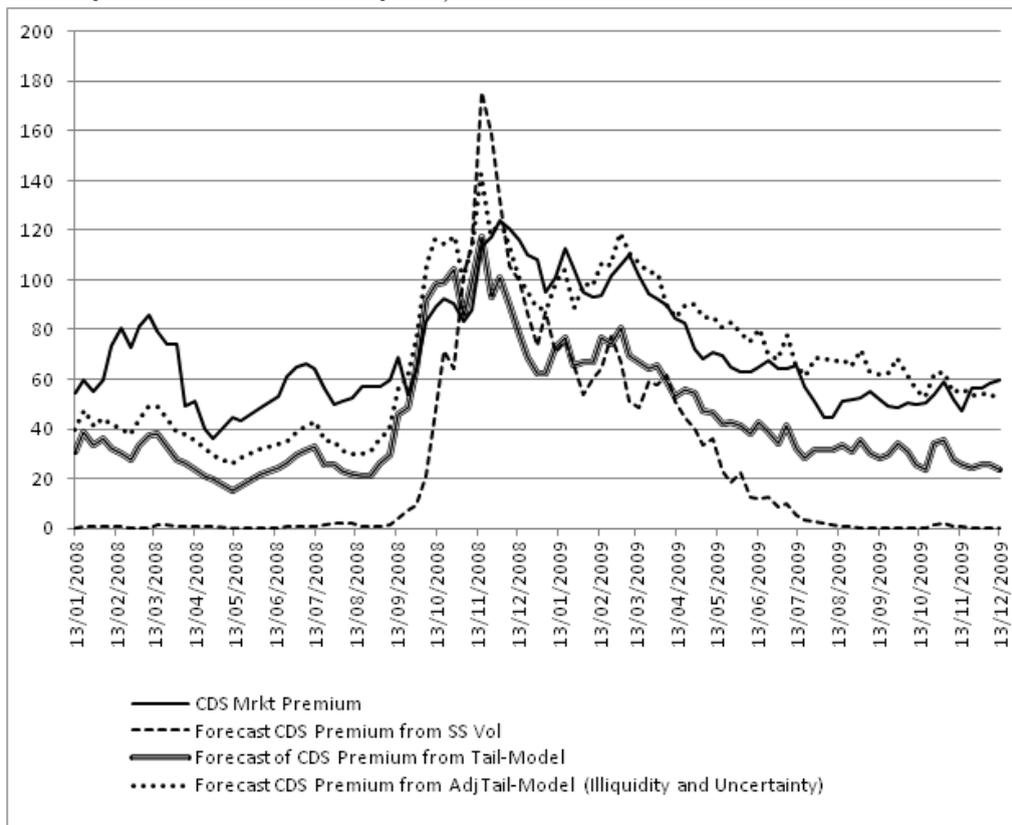


Figure 20: Comparison of Average Out-of-Sample Forecasts:
 CDS Market Premium, CDS Premium from Merton Model calibrated on SS volatility,
 CDS Premium from Tail-Model using $dCDS/d\sigma^E$,
 and CDS Premium from Tail-Model adjusted for Illiquidity and Uncertainty
 (Average values across 40 U.S. investment-grade firms; Rolling forecasts;
 In-sample period: 117 weeks starting with 1 Jan 2005 - 25 Mar 2007;
 Out-of-sample period: 1 April 2007 - 31 December 2009;
 Forecasted CDS premia are measured in basis points.)

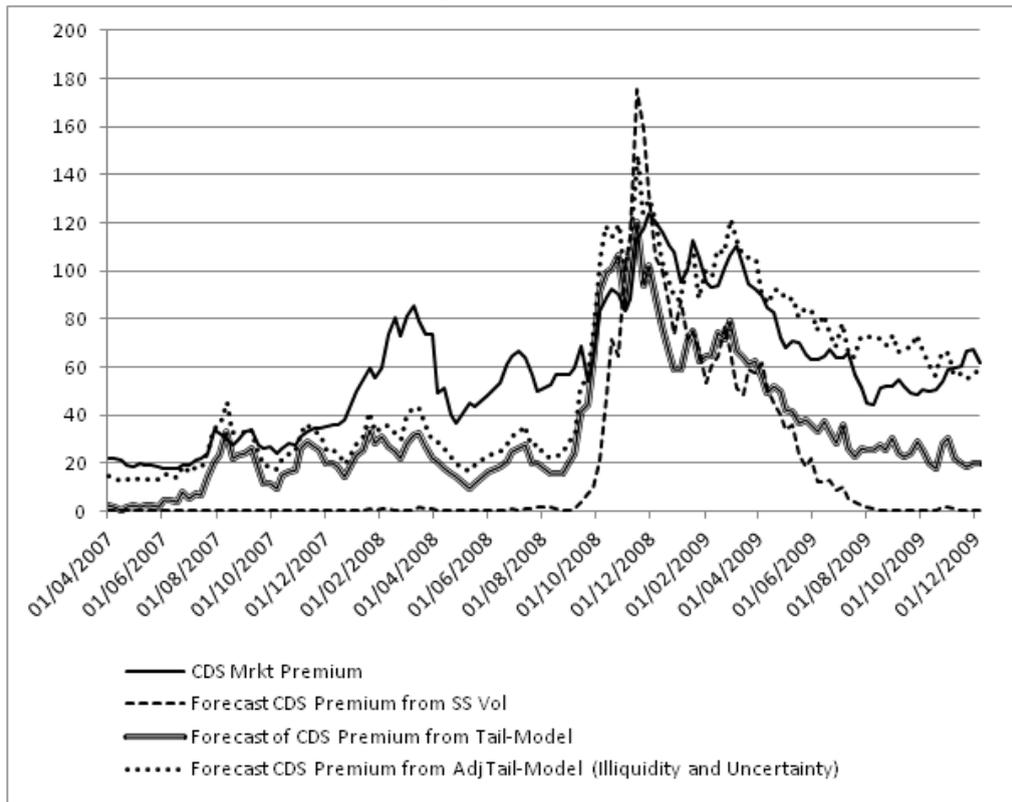


Figure 21: Scatter-Plot of CDS Premium Errors from Tail-Model against Residuals from Auxiliary Panel Regression (Eq. 4)

*(40 U.S. investment-grade firms; Sample: Jan 2005 - Dec 2009; Weekly frequency;
Panel residuals and tail-model CDS premium errors are measured in basis points.)*

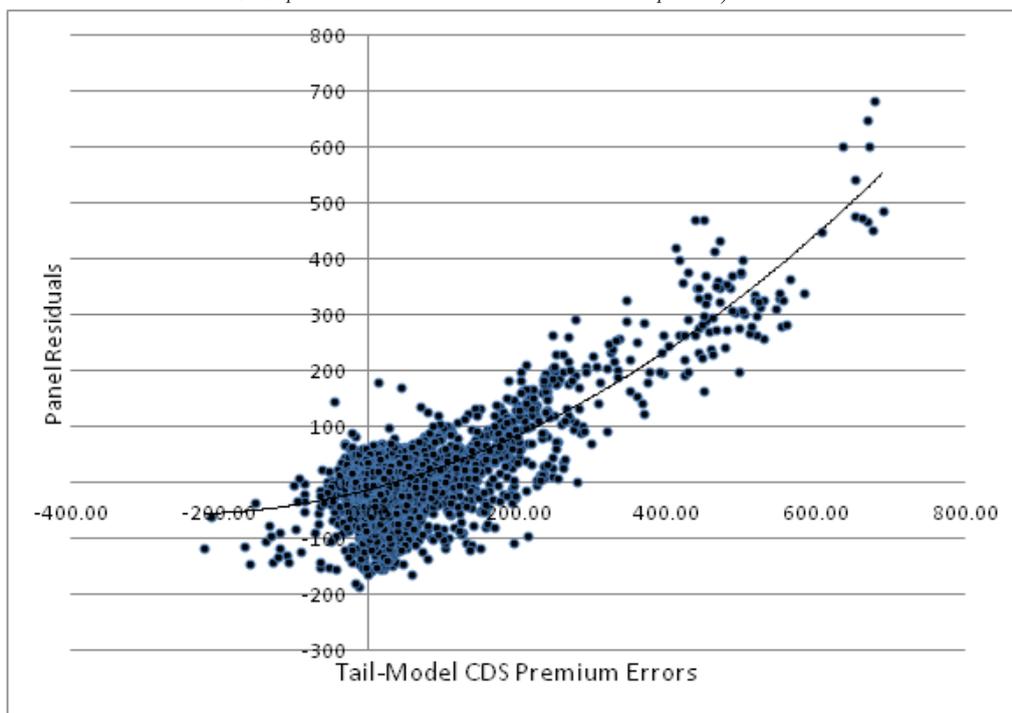


Figure 22: Comparison of Average Out-of-Sample Forecasts:
 CDS Market Premium, CDS Premium from Reduced-form Model (Auxiliary Panel Regression - Eq.(4)), CDS premium from Reduced-form model with Illiquidity and Uncertainty, and CDS Premium from Tail-Model adjusted for Illiquidity and Uncertainty

(Average values across 40 U.S. investment-grade firms; Rolling forecasts;

In-sample period: 157 weeks starting with 1 Jan 2005 - 6 Jan 2008;

Out-of-sample period: 13 January 2008 - 31 December 2009;

Forecasted CDS premia are measured in basis points.)

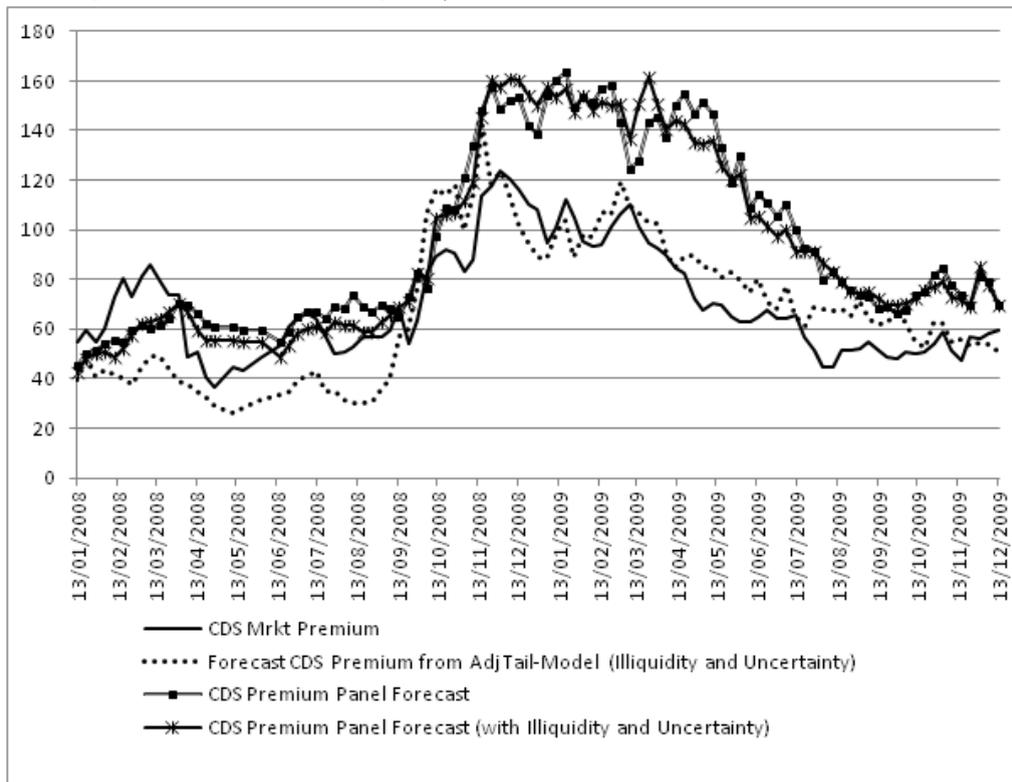


Figure 23: Comparison of Average Out-of-Sample Forecasts:
 CDS Market Premium, CDS Premium from Reduced-form Model (Auxiliary Panel Regression (Eq.(4))), CDS Premium from Reduced-form model with Illiquidity and Uncertainty, and CDS Premium from Tail-model adjusted for Illiquidity and Uncertainty

(Average values across 40 U.S. investment-grade firms; Rolling forecasts;

In-sample period: 117 weeks starting with 1 Jan 2005 - 25 Mar 2007;

Out-of-sample period: 1 April 2007 - 31 December 2009;

Forecasted CDS premia are measured in basis points.)

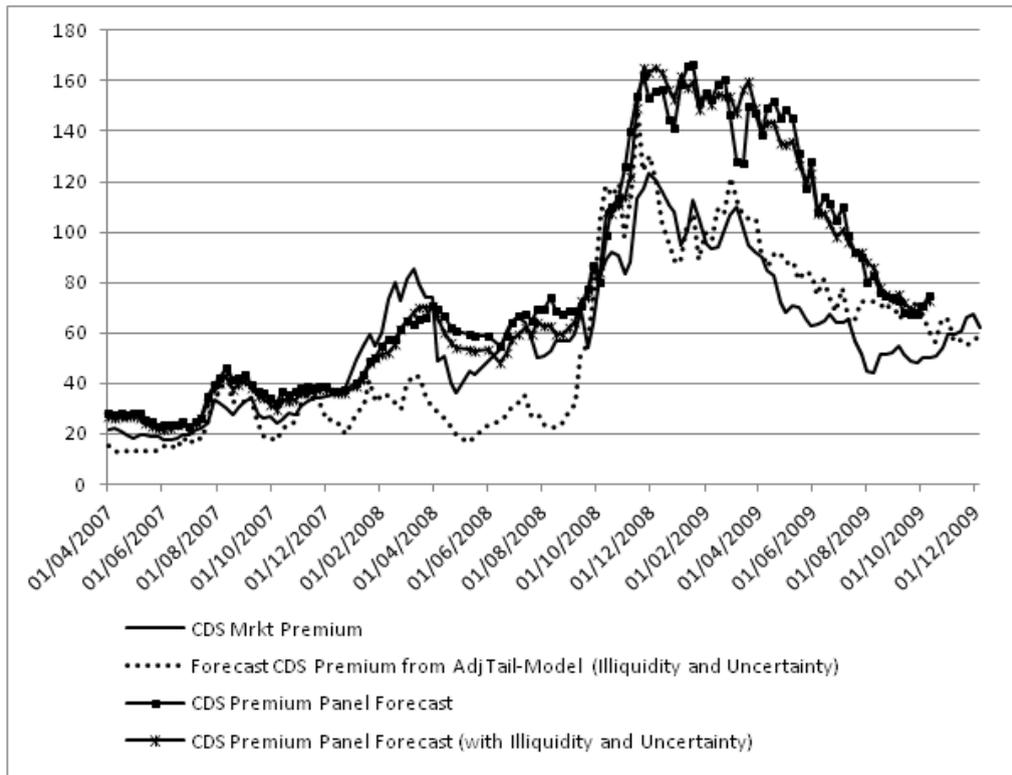


Table 1: Theoretical Values of CDS Premia and $dCDS/d\sigma^E$ from Merton Model

(The structural model is calibrated on the following assumptions:

- CDS contract written on a firm with asset value 100 and with 5-years maturity;
- Continuous interest rate r set equal to 5%;
- Target level of leverage set equal to the current level)

Leverage	Asset Volatility	CDS Premium (bp)	Equity Volatility	d CDS Premium/d EqVol
0.05	0.2	0	0.211	
0.05	0.25	0	0.263	0
0.05	0.3	0	0.316	0
0.05	0.35	0.1	0.368	0
0.05	0.4	0.8	0.421	0.1
0.05	0.45	3.5	0.474	0.5
0.05	0.5	10.5	0.526	1.3
0.05	0.55	24.3	0.579	2.6
0.05	0.6	47.4	0.631	4.4
0.05	0.65	81.6	0.683	6.6
0.15	0.2	0	0.235	
0.15	0.25	0.3	0.294	0
0.15	0.3	2.4	0.353	0.4
0.15	0.35	10.1	0.411	1.3
0.15	0.4	27.5	0.469	3
0.15	0.45	57.8	0.526	5.3
0.15	0.5	102.3	0.583	7.8
0.15	0.55	161.7	0.64	10.5
0.15	0.6	235.7	0.696	13.2
0.15	0.65	323.9	0.752	15.7
0.3	0.2	1.8	0.286	
0.3	0.25	11.6	0.356	1.4
0.3	0.3	36	0.424	3.6
0.3	0.35	77.9	0.491	6.3
0.3	0.4	137.1	0.557	9
0.3	0.45	212.6	0.622	11.6
0.3	0.5	302.8	0.687	13.9
0.3	0.55	406.5	0.751	16
0.3	0.6	522.7	0.817	17.8
0.3	0.65	650.7	0.883	19.4

Table 2: Average Estimates of $dCDS/d\sigma^E$ obtained from Cointegration Analysis on Individual Firms' CDS Premia and Equity Volatility

(Average results across 40 Investment Grade Firms are reported;

Pre-Crash is the period from 1/1/2005 to 25/3/2007;

Post-Crash is the period from 1/4/2007 to 27/12/2009)

	All Period	Pre-Crash Period	Post-Crash Period
Number of Firms	40	40	40
Number of Firms with $dCDS/d\sigma^E > 0$	40	29	40
Mean $dCDS/d\sigma^E$	3.152	0.643	3.471
Median $dCDS/d\sigma^E$	2.042	0.426	2.399
Std Dev $dCDS/d\sigma^E$	2.521	0.875	2.785
Min $dCDS/d\sigma^E$	0.713	-0.680	1.033
Max $dCDS/d\sigma^E$	10.170	3.025	10.858
Range	9.457	3.704	9.825
First Quartile $dCDS/d\sigma^E$	1.508	-0.005	1.597
Third Quartile $dCDS/d\sigma^E$	3.880	1.050	3.865
Inter-Quartile Range	2.372	1.055	2.268

Table 3: Average Estimates of $dCDS/d\sigma^E$ obtained from Cointegration Analysis on Individual Firms' CDS Premia and Equity Volatility, conditional on VIX Bands

(Average results across 40 Investment Grade Firms are reported)

VIX band	VIX < 13	13 < VIX < 15	15 < VIX < 20	20 < VIX < 25	25 < VIX < 30	30 < VIX < 50	50 < VIX < 60	VIX > 60
Summary Statistics for $dCDS/d\sigma^E$								
Tot Num. of Firms	40	40	40	40	40	40	40	40
Num. of Firm with $dCDS/d\sigma^E > 0$	29	27	31	32	35	37	35	35
Mean $dCDS/d\sigma^E$	0.557	0.442	0.872	1.593	2.155	2.299	0.844	4.808
Median $dCDS/d\sigma^E$	0.471	0.307	0.672	0.759	1.501	2.022	1.292	2.961
Std $dCDS/d\sigma^E$	1.091	0.837	1.638	2.652	2.686	2.693	5.801	7.084
Min $dCDS/d\sigma^E$	-1.288	-1.753	-1.439	-1.409	-4.309	-4.081	-29.649	-8.331
Max $dCDS/d\sigma^E$	4.350	2.997	7.866	10.489	10.601	9.752	11.409	33.214
Range	5.638	4.750	9.305	11.898	14.910	13.833	41.058	41.545
1st Quartile $dCDS/d\sigma^E$	-0.110	-0.076	0.061	0.069	0.694	1.000	0.593	0.846
3rd Quartile $dCDS/d\sigma^E$	1.169	1.033	0.963	2.008	3.130	2.955	2.373	7.162
Inter-Quartile Range	1.279	1.109	0.902	1.938	2.440	1.955	1.781	6.316

Table 4: Auxiliary Panel Regression of CDS Premia on:
Equity Volatility Variables, Interest Rate and Leverage (All-Sample)

Dependent Variable: CDS Market Premia (measured in basis points)

Method: Panel Least Squares

Sample (adjusted): 1/1/2005 - 27/12/2009

Periods (weeks) included: 257

Cross-sections (firms) included: 40

Total panel (balanced) observations: 10280

Firm clustered standard errors and covariance are estimated (d. f. corrected)

Second-stage Panel Regression. The equity volatility variable is the orthogonal component obtained as residual from a first-stage panel regression of firm's equity volatility on leverage and S&P 500 Index volatility

Variable	Coeff. Estimate	Std Error	t-Value	p-value
Intercept	41.5832	4.0824	10.19	<0.0001
Lev Ratio	138.8511	5.7952	23.96	<0.0001
Int. Rate	-11.9742	0.7903	-15.15	<0.0001
S&P500 Eq. Vol.	1.9437	0.0583	33.32	<0.0001
Equity Volatility	0.7078	0.1318	5.37	<0.0001
Eq. Vol. x Dummy VIX 13	0.0328	0.2398	0.14	0.8913
Eq. Vol. x Dummy VIX 15	0.1680	0.2467	0.68	0.4859
Eq. Vol. x Dummy VIX 20	2.2133	0.1877	11.79	<0.0001
Eq. Vol. x Dummy VIX 25	2.7876	0.2029	13.74	<0.0001
Eq. Vol. x Dummy VIX 30	4.5889	0.1710	26.83	<0.0001
Eq. Vol. x Dummy VIX 50	4.4992	\hat{A} 0.2098	\hat{A} 21.44	<0.0001
Adj. R-squared	0.5667			

Table 5: Cross-Sectional Regression - 19 March 2006

Dependent Variable: Tail-Model Premium Error =
CDS Market Premium - CDS Premium implied by Tail-Model

Cross-sections included: 40 IG Firms;

White standard errors are estimated;

CDS Res Illiquidity =

Orthogonal component of CDS bid-ask spread to past CDS premium and current equity volatility;

Earnings' Uncertainty = Dispersion of analysts' forecasts on the firm's earnings.

<i>Specification 1</i>	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob. \hat{A} \hat{A}
Constant	14.7477		3.6877	4.00	0.0003
Earnings Uncertainty	-2.3854	Not Sign.	13.8649	-0.17	0.8644
CDS Res Illiquidity	1.7958	Not Sign.	3.6784	0.49	0.6285
R-squared	0.007				
Adjusted R-squared	-0.050				
<i>Specification 2</i>	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob.
Constant	13.6684		7.8310	1.75	0.0899
Earnings Uncertainty	-3.2605	Not Sign.	15.1283	-0.22	0.8306
Equity Vol.	0.0599	Not Sign.	0.3821	0.16	0.8763
CDS Res Illiquidity	1.8868	Not Sign.	3.7757	0.50	0.6205
R-squared	0.008				
Adjusted R-squared	-0.078				

Table 6: Cross-Sectional Regression - 19 March 2006
 Dependent Variable: Implied Volatility Error from Tail-Model=
 Volatility implied from CDS Market Premium - Volatility implied from Tail-Model

Cross-sections included: 40 IG Firms;

White standard errors are estimated;

CDS Res Illiquidity =

Orthogonal component of CDS bid-ask spread to past CDS premium and current equity volatility;

Earnings' Uncertainty = Dispersion of analysts' forecasts on the firm's earnings.

<i>Specification 1</i>	Coefficient	Econ. Sign.	Std. Error	<i>t-Statistic</i>	Prob.
Constant	0.0367		0.0073	5.01	<0.0001
Earnings Uncertainty	-0.0003	Not Sign.	0.0275	-0.01	0.9912
CDS Res Illiquidity	0.0041	Not Sign.	0.0073	0.57	0.5746
R-squared	0.009				
Adjusted R-squared	-0.047				
<i>Specification 2</i>	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob.
Constant	0.0307		0.0155	1.98	0.056
Earnings Uncertainty	-0.0052	Not Sign.	0.0300	-0.17	0.8642
Equity Vol.	0.0003	Not Sign.	0.0008	0.44	0.6628
CDS Res Illiquidity	0.0046	Not Sign.	0.0075	0.62	0.5387
R-squared	0.015				
Adjusted R-squared	-0.072				

Table 7: Cross-Sectional Regression - 21 Sept 2008
 Dependent Variable: Tail-Model Premium Error =
 CDS Market Premium - CDS Premium implied by Tail-Model

Cross-sections included: 40 IG Firms;

White standard errors are estimated;

CDS Res Illiquidity =

Orthogonal component of CDS bid-ask spread to past CDS premium and current equity volatility.

Earnings' Uncertainty = Dispersion of analysts' forecasts on the firm's earnings; Significant Variables (at 1% C.L.) in bold.

<i>Specification 1</i>	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob.
Constant	5.1924		8.6645	0.60	0.5536
Earnings Uncertainty	74.5472	0.7433	17.8937	4.17	0.0002
CDS Res Illiquidity	22.0915	0.3632	7.8359	2.82	0.0079
R-squared	0.510				
Adjusted R-squared	0.482				
<i>Specification 2</i>	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob.
Constant	-56.3862		18.4164	-3.06	0.0043
Earnings Uncertainty	37.0120	0.3690	18.4945	2.00	0.0534
Equity Vol.	1.9362	0.3996	0.5297	3.66	0.0009
CDS Res Illiquidity	27.7207	0.4557	6.9098	4.01	0.0003
R-squared	0.648				
Adjusted R-squared	0.617				

Table 8: Cross-Sectional Regression - 21 Sept 2008

Dependent Variable: Implied Volatility Error from Tail-Model=
Volatility implied from CDS Market Premium - Volatility implied from Tail-Model

Cross-sections included: 40 IG Firms;

White standard errors are estimated;

CDS Res Illiquidity =

Orthogonal component of CDS bid-ask spread to past CDS premium and current equity volatility;

Earning Uncertainty = Dispersion of analysts' forecasts on the firm's earnings; Significant Variables (at 1% C.L.) in bold.

<i>Specification 1</i>	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob.
Constant	0.0001		0.0088	0.01	0.9883
Earnings Uncertainty	0.0732	0.9450	0.0182	4.02	0.0003
CDS Res Illiquidity	0.0207	0.4404	0.0080	2.59	0.0138
R-squared	0.484				
Adjusted R-squared	0.454				

<i>Specification 2</i>	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob.
Constant	-0.0674		0.0182	-3.71	0.0007
Earnings Uncertainty	0.0321	0.4138	0.0182	1.76	0.0876
Equity Vol.	0.0021	0.5670	0.0005	4.07	0.0003
CDS Res Illiquidity	0.0269	0.5717	0.0068	3.94	0.0004
R-squared	0.653				
Adjusted R-squared	0.622				

Table 9: Time-Series Regression - All Sample (Jan 2005 - Dec 2009)

Dependent Variable: Average Implied Volatility Error from Tail-Model=

Average Volatility implied from CDS Market Premia - Average Volatility implied from Tail-Model

Sample: 1/01/2005 - 27/12/2009 - Obs:257;

Newey-West HAC Standard Errors and Covariance (lag truncation=4);

CDS Res Illiquidity =

Residuals from a regression of CDS bid-ask spread on past CDS premium and current equity volatility; Significant Variables (at 1% C.L.) in bold.

	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob. $\hat{A} \hat{A}$
Constant	0.0272		0.0024	11.29	<0.0001
VIX	0.0001	Not Sign.	0.0001	1.38	0.1685
CDS Res Illiquidity	0.0153	0.4240	0.0021	7.45	<0.0001
R-squared	0.182				
Adjusted R-squared	0.175				

Table 10: Time-Series Regression - All Sample (Jan 2005 - Dec 2009)

Dependent Variable: Average Tail-Model Premium Error

Average CDS Market Premium - Average CDS Premium implied from Tail-Model

Sample: 1/01/2005 - 27/12/2009 - Obs:257;

Newey-West HAC Standard Errors and Covariance (lag truncation=4);

CDS Res Illiquidity =

Residuals from a regression of CDS bid-ask spread on past CDS premium and current equity volatility; Significant Variables (at 1% C.L.) in bold.

	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob.
Intercept	-1.7015		2.1872	-0.78	0.4373
VIX	1.2121	0.6174	0.0880	13.78	<0.0001
CDS Res Illiquidity	15.6739	0.3761	1.8674	8.39	<0.0001
R-squared	0.492				
Adjusted R-squared	0.488				

Table 11: Panel Regression

Dependent Variable: Implied Volatility Error from Tail-Model=

Volatility implied from CDS Market Premia - Volatility implied from Tail-Model

Method: Panel Least Squares;

Sample: 1/01/2005 - 27/12/2009 (All Sample);

Periods (weeks) included: 257;

Cross-sections (firms) included: 40;

Total panel (unbalanced) observations: 10280;

White period standard errors and covariance (d. f. corrected);

CDS Res Illiquidity =

Residuals from a regression of CDS bid-ask spread on past CDS premium and current equity volatility;

Earning Uncertainty = Dispersion of analysts' forecasts on the firm's earnings;

Firms' Fixed Effects Included (Test for Fixed Effect: F-Stat= 178.78; (Pr>F)<0.0001); Significant Variables (at 1% C.L.) in bold.

	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob.
Constant	0.024		0.00250	9.53	<.0001
VIX	0.0001	0.0225	0.00003	2.79	0.0052
Earnings Uncertainty	0.0008	0.1072	0.00006	13.03	<.0001
CDS Res Illiquidity	0.0060	0.1284	0.00040	15.78	<.0001
Adjusted R-squared	0.369				
Constant	0.0093		0.0023	3.99	<.0001
Inv. Mkt. Cap	0.0039	0.5752	0.0012	34.02	<.0001
Earnings Uncertainty	0.0007	0.0895	0.00006	11.5	<.0001
CDS Res Illiquidity	0.0046	0.0969	0.0004	12.52	<.0001
Adjusted R-squared	0.436				

Table 12: Panel Regression

Dependent Variable: Tail-Model Premium Error=
CDS Market Premium - CDS Premium implied from Tail-Model

Method: Panel Least Squares;

Sample: 1/01/2005 27/12/2009 (All Sample);

Periods (weeks) included: 257;

Cross-sections (firms) included: 40;

Total panel (unbalanced) observations: 10280;

White period standard errors and covariance (d. f. corrected);

CDS Res Illiquidity =

Residuals from a regression of CDS bid-ask spread on past CDS premium and current equity volatility;

Earning Uncertainty = Dispersion of analysts' forecasts on the firm's earnings;

Firms' Fixed Effects Included. (Test for Fixed Effect: F-Stat= 108.58; (Pr>F)<0.0001); Firms' Fixed Effects Included (Test for Fixed Effect: F-Stat= 178.78; (Pr>F)<0.0001); Significant Variables (at 1% C.L.) in bold.

	Coefficient	Econ. Sign.	Std. Error	t-Statistic	Prob.
Constant	-7.3431		3.4055	-2.16	0.0311
VIX	1.1636	0.2290	0.0428	27.19	<.0001
Earnings Uncertainty	1.2681	0.1264	0.0861	14.73	<.0001
CDS Res Illiquidity	8.1983	0.1348	0.5167	15.87	<.0001
Adjusted R-squared	0.312				
Constant	-15.9093		3.0691	-5.18	<.0001
Inv. Mkt. Cap	7.9124	0.8986	0.1505	52.57	<.0001
Earnings Uncertainty	1.0712	0.1068	0.0790	13.57	<.0001
CDS Res Illiquidity	4.7649	0.0783	0.4758	10.01	<.0001
Adjusted R-squared	0.424				

Table 13: Pair-wise Correlation Matrix with Relative T-Statistics at 95% Confidence Level for VIX, CDS Residual Bid-Ask Spreads, Analysts' Earnings Forecast Dispersion and Inverse of Market Capitalization

(All Sample: Jan 2005 - Dec 2009)

	VIX	CDS Res Illiquidity	Earnings Uncertainty
CDS Res Illiquidity	-0.0371 (<i>t-stat = 0.0002</i>)		
Earnings Uncertainty	0.0394 (<i>t-stat < 0.0001</i>)	-0.0003 (<i>t-stat = 0.9794</i>)	
Inverse of Market Cap.	0.1859 (<i>t-stat < 0.0001</i>)	0.0686 (<i>t-stat < 0.0001</i>)	0.0078 (<i>t-stat = 0.4403</i>)

Table 14: Robustness Check:
 Panel Regression of CDS Premia on Equity Volatility Variables, Interest Rate, CDS Residual Bid-Ask Spread and Analysts' Earnings Forecast Dispersion

Dependent Variable: CDS Market Premia;

Method: Panel Least Squares;

Sample (adjusted): 1/1/2005 - 27/12/2009 (All Sample);

Periods (weeks) included: 257;

Cross-sections (firms) included: 40;

Total panel (balanced) observations: 10280;

Clustered standard errors and covariance at firms level are estimated (d. f. corrected);

CDS Res Illiquidity =

Residuals from a regression of CDS bid-ask spread on past CDS premium and current equity volatility;

Earning Uncertainty = Dispersion of analysts' forecasts on the firm's earnings.

Variable	Coeff. Estimate	t-Value	p-value
Constant	28.2751	6.94	<0.0001
Lev Ratio	136.4608	24.03	<0.0001
Int. Rate	-9.3289	-11.82	<0.0001
S&P500 Eq. Vol.	2.0974	36.20	<0.0001
Equity Volatility	0.8837	6.80	<0.0001
Eq. Vol. x Dummy VIX 13	0.1059	0.45	0.6523
Eq. Vol. x Dummy VIX 15	0.2179	0.90	0.3674
Eq. Vol. x Dummy VIX 20	2.1099	11.47	<0.0001
Eq. Vol. x Dummy VIX 25	2.6182	13.16	<0.0001
Eq. Vol. x Dummy VIX 30	4.2497	25.16	<0.0001
Eq. Vol. x Dummy VIX 50	4.0102	19.26	<0.0001
CDS Res. Bid-Ask Spread	6.8822	13.84	<0.0001
Earnings Forecast Dispersion	1.1691	14.85	<0.0001
<i>Adj. R-squared</i>	<i>0.5849</i>		

Table 15: Out-of-Sample Forecasting Performance of Different Methodologies for CDS Premia Estimation (Rolling In-Sample Window of 157 weeks)

Out-of-Sample Forecasts obtained from Merton Model calibrated on:

- Schaefer and Strebulaev (2008) Volatility,
 - $dCDS/d\sigma^E$ (Tail-Model),
 - $dCDS/d\sigma^E + \text{Adjustment for Illiquidity and Uncertainty (Adjusted Tail-Model)}$;
 - Forecasts obtained from Panel Regression including only Structural Variables (Panel Forecast);
 - Forecast obtained from Panel Regression including also Illiquidity and Uncertainty (Adjusted Panel Forecast).
- (In-sample Period 01/01/2005 - 06/01/2008; Out-of-sample Period 13/01/2008 - 27/12/2009)

	SS Model	Tail-Model	Adj. Tail-Model	Panel Forecast	Adj. Panel Forecast	Panel Forecast	Adj. Panel Forecast
<i>Summary Statistics for Absolute Forecast Errors</i>							
Mean (MAE)	48.073	26.524	14.345	28.920	28.920	28.920	29.212
Max	85.389	50.145	38.442	83.250	83.250	83.250	75.199
Min	7.588	0.640	0.807	0.216	0.216	0.216	0.765
Range	77.801	49.505	37.635	83.034	83.034	83.034	74.434
First quartile	38.532	20.999	6.517	15.473	15.473	15.473	12.469
Third quartile	56.323	32.082	20.920	40.990	40.990	40.990	43.771
Inter-quartile Diff	17.791	11.084	14.402	25.518	25.518	25.518	31.301
Std Dev	15.326	10.221	9.111	19.688	19.688	19.688	20.442
MSE	2543.506	806.936	287.942	1220.066	1220.066	1220.066	1266.961
RMSE	50.433	28.407	16.969	34.929	34.929	34.929	35.594
<i>Horse-Race MAE</i>	5	2	1	3	3	3	4
<i>Horse-Race RMSE</i>	5	2	1	3	3	3	4

Table 16: Out-of-Sample Forecasting Performance of Different Methodologies for CDS Premia Estimation (Rolling In-Sample Window of 117 weeks)

Out-of-Sample Forecasts obtained from Merton Model calibrated on:

- Schaefer and Strebulaev (2008) Volatility,
 - $dCDS/d\sigma^E$ (Tail-Model),
 - $dCDS/d\sigma^E$ + Adjustment for Illiquidity and Uncertainty (Adjusted Tail-Model);
 - Forecasts obtained from Panel Regression including only Structural Variables (Panel Forecast);
 - Forecast obtained from Panel Regression including also Illiquidity and Uncertainty (Adjusted Panel Forecast).
- (In-sample Period 01/01/2005 - 25/03/2008; Out-of-sample Period 01/04/2007 - 27/12/2009)

	SS Model	Tail-Model	Adj. Tail-Model	Panel Forecast	Adj. Panel Forecast	Panel Forecast	Adj. Panel Forecast
<i>Summary Statistics for Absolute Forecast Errors</i>							
Mean (MAE)	42.111	26.121	14.818	22.764	21.488	22.764	21.488
Max	85.389	55.615	47.704	80.752	66.993	80.752	66.993
Min	7.588	1.447	0.287	0.149	0.685	0.149	0.685
Range	77.801	54.168	47.417	80.603	66.308	80.603	66.308
First quartile	27.894	15.551	4.409	7.429	4.995	7.429	4.995
Third quartile	54.530	35.182	22.637	34.869	36.246	34.869	36.246
Inter-quartile Diff	26.636	19.631	18.228	27.439	31.251	27.439	31.251
Std Dev	16.917	12.794	11.693	20.011	19.759	20.011	19.759
MSE	2057.468	844.818	355.313	915.700	849.329	915.700	849.329
RMSE	45.359	29.066	18.850	30.261	29.143	30.261	29.143
Horse Race MAE	5	4	1	3	2	3	2
Horse Race RMSE	5	2	1	4	3	4	3

B The Theory of the Merton Model (1974)

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and $(W_t)_{0 \leq t \leq T}$ a Brownian motion defined on this space. The firm is represented by an asset value A which pays a payout rate δ . Merton model (1974) conjectures that the total value of a firm's asset A follow a log-normal diffusion process with constant growth rate μ^A and constant volatility σ^A :

$$dA_t = \mu^A A_t dt + \sigma^A A_t dW_t \quad (\text{B.1})$$

where $\mu^A = r - \delta$.

The firms' liabilities consist of risky debt B (with face value D and maturity T) and equity E . The firm's leverage L is defined as the ratio between the present value of debt promised payment D and the total value of the assets A . Thus, it is equal to: $L = \frac{De^{-rT}}{A}$, where r is the continuously compounded risk-free interest rate in the market. Firm's default occurs at date- T (maturity) if $A_T < D$.

Under the assumptions of the Black-Scholes (1973) model³², the Merton (1974) model prices equity and risky debt of a firm as contingent claims written on the firm's assets. Equity is priced as a European call option on the assets of the firm with strike price equal to the face value of debt D . The risky debt of the firm is instead evaluated as a short position on European put written on the firm's asset (with strike equal to the promised debt payment D) and a long position on a riskless bond. Therefore, according to the Black-Scholes pricing formula for non-dividend paying European call options, at time 0 the equity value E_0 is given by:

$$E_0 = C^{BS}(A_0, \sigma^A, D, r, T) = A_0 N(d_1) - De^{-rT} N(d_2) \quad (\text{B.2})$$

where $N(\cdot)$ is the cumulative function for the standard Normal distribution,

$$d_1 = \frac{\ln(\frac{A_0}{De^{-rT}}) + \frac{\sigma^A \sqrt{T}}{2}}{\sigma^A \sqrt{T}} = \frac{-\ln(L) + \frac{\sigma^A \sqrt{T}}{2}}{\sigma^A \sqrt{T}}$$

and $d_2 = d_1 - \sigma^A \sqrt{T}$.

The sensitivity (first derivative) of equity to firm's total assets value is determined by the call option delta: $N(d_1) = \Delta_C$.

At time 0, the debt value B_0 is given by the difference between the total assets' value and the equity value:

$$B_0 = A_0 - E_0 \quad (\text{B.3})$$

Using Equations (B.2) and (B.3) we obtain:

$$B_0 = De^{-rT} N(d_2) + A_0 N(-d_1) \quad (\text{B.4})$$

³²The Assumptions behind Black-Scholes model (1973) and Merton model (1974) are the following:

- Market are competitive and efficient: agents are price-takers and trading has no affect on prices;
- There are no transaction costs;
- Agents trade continuously;
- Agents have unlimited access to short-selling and assets are indivisible;
- There are no bankruptcy costs in case of firm's default;
- There are no corporate taxes or tax advantages from issuing debt;
- Agents can borrow and lend at the same continuously compounded risk-free rate r ;
- The firm has issued only two kinds of claims: non-dividend paying equity and debt. Debt is a pure zero-coupon bond that pays at maturity T an amount D .

This implies:

$$B_0 = De^{-rT} - (De^{-rT}N(-d_2) - A_0N(-d_1)) = PV(D) - P^{BS}(A_0, \sigma^A, D, r, T) \quad (\text{B.5})$$

As previously mentioned, the debt value at time 0 is equal to the present value of a long position on a riskless bond with face value D plus the value of a short position on a put option (derived from the Black-Scholes pricing formula for non-dividends paying European put options).

Thus, the credit spread on the risky bond at time t is given by: $s_t = -\frac{1}{T-t} \ln(\frac{B_t}{D}) - r$.

Equation (B.4) can be used to calculate the sensitivity (first derivative) of risky debt to assets' value which is given by the delta of the put option: $N(-d_1) = \Delta_P$.

The sensitivity of debt to equity h is then given by:

$$h = \frac{\partial B}{\partial E} = \frac{\frac{\partial B}{\partial A}}{\frac{\partial E}{\partial A}} = \frac{N(-d_1)}{N(d_1)} = \frac{1}{\Delta_c} - 1 \quad (\text{B.6})$$

Therefore it depends on the delta of a European call option written on the firm's assets with exercise price equal to the face value of debt. The debt-to-equity elasticity (H hedge ratio) is obtained as:

$$H = \left(\frac{\partial B}{\partial E}\right)\left(\frac{E}{B}\right) = h\left(\frac{1}{L} - 1\right) \quad (\text{B.7})$$

Two common methodologies to calibrate the Merton model are the one of Vassalou and Xing (2004) - henceforth VX Methodology - and the one implemented by Schaefer and Strebulaev (2008) - henceforth SS Methodology. The VX methodology requires the knowledge of the outstanding debt of the firm, the equity value, and the equity volatility³³ in order to estimate the value and volatility of the firm's assets from a system of two non-linear equations. Following Vassalou and Xing (2004), we recall Equation (B.2) and notice that since equity is a function of assets' value, it is possible to apply Ito's Lemma to determine the instantaneous volatility of equity σ^E from total assets' volatility σ^A (Jones et al, 1984).

$$dE_t = df(A_t, t) = \left(\frac{\partial E}{\partial t} + \mu_A A_t \frac{\partial E}{\partial A} + \frac{\sigma^A{}^2}{2} A_t^2 \frac{\partial^2 E}{\partial A^2}\right)dt + (\sigma^A A_t \frac{\partial E}{\partial A})dW_t. \quad (\text{B.8})$$

It follows:

$$E_0 \sigma^E = A_0 \sigma^A \frac{\partial E}{\partial A} = A_0 \sigma^A N(d_1). \quad (\text{B.9})$$

and

$$\sigma^E = \frac{\sigma^A A_0 N(d_1)}{E_0}. \quad (\text{B.10})$$

Equations (B.2) and (B.10) represent a system of two equations in two unknowns (A_0 and σ^A). Therefore we can determine the unknowns by solving the non-linear equations. In practice, we adopt a recursive procedure (the so-called KMV method; see also Crosbie and Bohn, 2003, and Bharath

³³Typically, equity volatility is estimated from historical annualized volatility of equity daily log returns; the firm's equity value is obtained as a product of the firm's equity price and the number of its outstanding shares (i.e. the firm's market capitalization); and the outstanding amount of debt can be obtained as the book value of the firm's current debt plus half of its long-term debt value.

and Shumway, 2004) that involves inverting the Black-Scholes formula³⁴.

The SS Methodology estimates asset volatility in a “more direct, model-free approach that is based only on observables” and “recognizes that debt bears some asset risk and that equity and debt covary” (Schaefer and Strebulaev, 2008). The methodology requires an estimation of the asset volatility for each firm i at time t as square root of:

$$\sigma_{i,t}^A{}^2 = (1 - L_{i,t})\sigma_{i,t}^E{}^2 + L_{i,t}\sigma_{i,t}^D{}^2 + 2(1 - L_{i,t})L_{i,t}\sigma_{i,t}^{ED} \quad (\text{B.11})$$

$\sigma_{i,t}^D$ is the time t unconditional volatility of firm i debt - estimated as the historical annualized volatility of debt log returns; $\sigma_{i,t}^E$ is the time t unconditional volatility of firm i equity - estimated as the historical annualized volatility of equity log returns; $\sigma_{i,t}^{ED}$ is the time t covariance between firm i debt and equity - estimated as the historical annualized covariance between equity and debt returns; and $L_{i,t}$ is the leverage ratio of firm i at time t . Once A and σ^A are estimated, then it is possible to estimate also $N(d_1)$, the debt-to-equity hedge ratio H and the credit spread implied by the Merton (1974) model.

³⁴Crosbie et al (2003) explain that the model linking equity and asset volatility, described by the system of Equations (B.2) and (B.10), holds only instantaneously. In practice the market leverage moves around in a substantial way and the system does not provide reasonable results. Instead of using the instantaneous relationships given by Equations (B.2) and (B.10), we follow Crosbie et al (2003) and estimate the model using a more complex iterative procedure to solve for the asset volatility. Crosbie et al (2003) describe it as a procedure that “uses an initial guess of the volatility to determine the asset value and to de-lever the equity returns. The volatility of the resulting asset returns is used as the input to the next iteration of the procedure that in turn determines a new set of asset values and hence a new series of asset returns. The procedure continues in this manner until it converges. This usually takes no more than a handful of iterations if a reasonable starting point is used”.

C Modified Merton Model and Calibration Procedure

In the paper the Merton (1974) model is slightly modified. A simplified version of the model by Collin-Dufresne and Goldstein (2001) is instead used (CDG). The original CDG model introduces stationary mean-reverting leverage ratio. In the CDG model firms adjust their capital structure (rather than keeping it fixed as in the original Merton model) in order to reflect changes in assets value. This ensures that the leverage ratio always reverts to a target value. In our simplified version of the CDG model, the leverage ratio is assumed to be fixed at its target value (or, equivalently, when the firm's assets value A changes, the leverage L is assumed to adjust immediately to its target value through an adjustment in the capital structure K).

At any time t , the current level of leverage $L_t = \frac{K_t}{A_t}$ is considered the target leverage level.

Therefore, while in the original Merton (1974) model $K_t = K \forall t \in [0, T]$, here we assume:

$$\frac{dK_t}{K_t} = (r - \delta)dt,$$

i.e. changes in assets value due to the net firm's payouts trigger an immediate adjustment in firm's capital structure to keep the leverage ratio invariant. As a consequence, the firm's credit spread is unaffected by the firm's payout rate (Collin-Dufresne and Goldstein (2001) obtain this result numerically through their model calibration).

We calibrate the structural model to parameters that match the following data: the current leverage ratio (given by book value of the firm's liabilities divided by the sum of market capitalization of the firm and book value of its liabilities), the riskless interest rate (set equal to the current 3-months T-Bill yield), the payout rate set equal to 2% (as explained above, the payout rate δ is irrelevant for credit spreads at the target leverage ratio), and the sensitivity of CDS premia to equity volatility estimated across all 40 firms over the historical period of 2005-2009. The estimation of $dCDS/d\sigma^E$ is obtained from a panel regression of CDS premia on leverage, interest rate, equity market volatility (approximated by the exponentially-weighted moving average of S&P volatility) and firm's equity volatility (conditional on VIX levels). The firm's equity volatility used in the panel regression is the orthogonal component of firm's equity volatility to equity market volatility and leverage. $dCDS/d\sigma^E$ is given by the sensitivity of CDS premia to the orthogonal firm's equity volatility *plus* the sensitivity of CDS spread to equity market volatility (see Section 4 for further details).

D Simultaneous Effects of Leverage on Assets Volatility and CDS Premia

The empirical results of the cointegration analysis in Table 2 reveal that over the pre-crash period for 25% of the firms in the sample an increase in equity volatility reduces (rather than increases) the CDS premium, thus ($dCDS/d\sigma^E < 0$). How can we explain this puzzling result?

It is reasonable to assume that over time $dCDS/dL > 0$ (i.e. a higher leverage ratio triggers a higher CDS premium). The positive effect of leverage on CDS premia over time is observed also in the results of the auxiliary panel regression - Equation (4) (see Table 4). If $dCDS/dL > 0$, then we can easily see that:

$$\frac{dCDS}{d\sigma^E} = \frac{dCDS}{dL} - \frac{1}{(d\sigma^E/dL)} > 0$$

if and only if $d\sigma^E/dL > 0$ (i.e. a higher leverage ratio triggers higher equity volatility); while

$$\frac{dCDS}{d\sigma^E} = \frac{dCDS}{dL} - \frac{1}{(d\sigma^E/dL)} < 0$$

if and only if $d\sigma^E/dL < 0$ (i.e. a higher leverage ratio triggers lower equity volatility).

To collect some heuristic evidence on this argument, we estimate $d\sigma^E/dL$ as slope coefficient of a panel regressions of equity volatility on leverage³⁵ (controlling also for market volatility), separately over the pre-crash period (Jan 2005 - June 2007) and the post-crash period (July 2007 - Dec 2009). We find that before the crisis $d\sigma^E/dL < 0$, while during the crisis $d\sigma^E/dL > 0$. In both cases the slope coefficients are statistically significant. Therefore, when higher leverage increases the CDS premium but simultaneously decreases firm's assets volatility, it is possible to observe negative values for the CDS premia sensitivity to equity volatility.

Leverage can have different cross-sectional and time-series effects on CDS premia and implied asset volatility. Let us consider the following example, displayed by Figure D.1. To illustrate this example we also refer to the results of the theoretical estimation of CDS premia (from Merton model) in Table 1. First, we simulate values for asset volatility using the tail-model for a number of firms with different levels of leverage (and we obtain the blue curve in Figure D.1). Second, we look at a specific investment-grade firm (let us call it firm J) with initial leverage ratio of 15% and asset volatility 48%.

The values of leverage ratio and asset volatility for firm J are represented by Point 1 on the blue curve:

Point 1: (Lev=0.15, AssVol=0.48)

The CDS premium for this firm should be around 80-90 bps (from an approximation with linear

³⁵Leverage is defined as the ratio between the book value of liabilities and the sum of the market value of equity and the book value of liabilities. The book value of liabilities and the number of shares outstanding change at a much lower frequency than the share price. Therefore, an increase in leverage over time is mainly triggered by a decrease in assets value, via a drop in share price.

interpolation using values provided by Table 1).

Let us suppose that in the following week, characterized by higher market turbulence, the asset volatilities of all firms increase by 10%, while their leverage ratios stay constant. Firm J will then move from the initial Point 1 to Point 2 on the upward-shifted volatility smile where its asset volatility is around 0.53 (red curve in Figure D.1):

Point 2: (Lev=0.15, AssVol=0.53)

The CDS premium of firm J should now be approximately equal to 132 bps (from linear interpolation of values in Table 1), that is 65% higher than in Point 1.

If instead during this week the asset volatility of firm J increases by 10% and also its leverage ratio increases by (say) 20%, firm J first moves from Point 1 to Point 2 on the red curve (because of the increase in volatility), and then from Point 2 to Point 3 (because of the increase in leverage):

Point 3: (Lev=0.18, AssVol= 0.50).

The net increase in asset volatility of firm J would be around 4% because the negative effect of leverage neutralizes part of the initial 10% increase in asset volatility. At Point 3 the CDS premium for firm J would be around 140 bps, that is 75% larger than in Point 1, but only 6% larger than in Point 2.

The contribution of leverage to the increase in the CDS premium appears nearly irrelevant, if compared to the contribution of asset volatility (despite we assume a 20% increase in leverage, which is double the increase in volatility of 10%).

To conclude, from this numerical example we observe that the negative effect of higher leverage on the implied volatility *in cross-section* (i.e. “sliding down” the volatility smile) is neutralized by the positive effect of an increase in general market volatility *over the two consecutive weeks*. In addition, the positive effect of the increase in leverage on CDS premia *over the two consecutive weeks* is less strong than the effect of the (even lower) increase in asset volatility.

Figure D.1: Simulation of Theoretical Asset Volatility Levels from Merton Model, given different Levels of Leverage

(The structural model is calibrated on the following assumptions:

- CDS contract written on a firm with asset value 100 and with 5-years maturity;
- Continuous interest rate r set equal to 5%;
- Target level of leverage set equal to the current level)

